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Dynamics and stability of moving fronts of water evaporation in a porous medium

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ABSTRACT

We study stability of non-steady plane water evaporation fronts, arising in vertical flows with phase transition in horizontally extended domains of a porous medium. Isothermal motions in the non-wettable horizontal porous medium when a water layer is located over a vapor one is considered and homogeneous capillary forces on the phase transition front are taken into consideration. It is shown that motion of the mobile phase transition fronts depend on their initial position relative to bifurcation curves of the stationary fronts. The four domains in the parameter space are picked out where behavior of linear perturbations of the mobile fronts are different: in the first domain all perturbations decrease, in the second one only long-wave perturbations grow, in the third domain only short-wave perturbations grow and in the fourth one all perturbations grow. Dynamics of localized finite amplitude perturbations of the moving phase transition front are also studied numerically. It is established that their evolution is mainly influenced by the initial position of the perturbed front relative to the mentioned stability/instability domains.

1. Introduction

We consider the model describing, for example, filtration processes in natural massifs, having contact with mines, tunnels and other constructions. The functioning of such engineering systems is accompanied by heat and mass exchange between the construction and surrounding rock [1]. Artificial ventilation makes it possible to keep the micro-climate, necessary for exploitation. Ventilation is accompanied by evaporation from a ceiling of the construction while the ground water moves downwards under the action of gravity or pressure in the water horizon. The water can enter the underground construction either in liquid or vapor states. If the surrounding rock has relatively low permeability it is natural to assume that the underground water moving towards the ceiling of the construction evaporates somewhere in a porous space and diffuses into the underground construction as a vapor. In this case a region saturated with a blend of vapor and air arises between the free space and water saturated region.

It is well known that for immiscible fluids the configuration with the heavier fluid overlying the lighter one is always subjected to the Rayleigh–Taylor instability even in a porous medium having an arbitrary small permeability [2]. The interface separating immiscible fluids has to deform in a way to prevent both fluids from blending, rather than the phase transition front, which deformations keep the temperature and pressure values for it on the Clapeyron curve of phase equilibrium. This difference in physical properties of the interface and the phase transition front explains the possibility of existence of a stable configuration with phase transitions even in the case when the heavier fluid overlies the lighter one in the porous medium. For the first time the stability of such a configuration was considered in [2] (see also [3–5]) where an example of a geothermal system is treated and existence of two domains saturated by motionless water and vapor is supposed.

As it was mentioned, in the case under consideration, the domain is formed saturated with a blend of vapor and air and located under the water saturated domain. The arising stationary evaporation front separating these two domains can be either stable or non-stable [6,7]. In [6] by the method of normal forms stability of the vertical flow was studied with the stationary phase transition front. This stability analysis concerns the behavior of infinitely small harmonic perturbations of the front. It was shown that there exist two types of instability. The first one is newly found and implies first destabilization of the mode with zero wave

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Nomenclature

Latin symbols

a	thermal diffusivity [m^2 s^{-1}]
q	specific heat of phase transition [J kg^{-1}]
$\mathbf{w}$	velocity [m s^{-1}]
$\mathbf{n}$	vector of normal [m]
g	acceleration of gravity [m s^{-2}]
h	location parameter of the interface [m]
k	permeability [m^2]
L	thickness of the low permeable stratum [m]
m	porosity [1]
P	pressure [Pa]
D	diffusion coefficient [m^2 s^{-1}]
t	time [s]
T	temperature [K]
$\mathbf{V}$	velocity of the phase transition interface [m s^{-1}]
R	Clapeyron constant [J kg^{-1} K^{-1}]
z	vertical coordinate [m]
$Z(t)$	location of the moving phase transition front [m]
x	horizontal coordinate [m]

Greek symbols

ν [l]	humidity
η	perturbation of the interface [m]
$\varkappa$	wave number [m^{-1}]
κ [l]	dimensionless exponent for nonhomogeneous perturbations
λ	thermal conductivity [W m^{-1} K]
μ	viscosity [Pa s]
ρ	density [kg m^{-3}]
σ	spectral parameter [s^{-1}]

Subscripts

1	right ahead of the interface in the water saturated domain
2	right behind the interface in the vapor domain
n	normal
D	diffusion
v	vapor
w	water
a	air
c	capillary
0	boundary value at $z = 0$
$*$	at the phase transition front

Superscript

0	boundary value at $z = L$
$'$	perturbation

number (long-wave instability), and the second implies first destabilization of the mode with infinite wave number (short-wave instability). Analysis when the basic regime with the stationary plane phase transition front is subjected to finite localized perturbations requires the use of sophisticated numerical methods and along with the study of the basic physical effects allows to determine the bounds of application of fundamental physical results of [6] for the case of localized finite amplitude perturbations [8]. In [6,7] it is also shown that a narrow band of weakly unstable modes in some neighborhood of the instability threshold of the existing pair of phase transition fronts are described by the nonlinear diffusion KPP equation [9]. All studies were done for the case of a non-wettable rock.

In this paper we consider stability properties of moving phase transition fronts and the dynamics of their linear and nonlinear perturbations in the described system. The paper is organized as follows. In Section 2 we give the formulation of the problem. The forms of the solutions describing the vertical flow with stationary and moving front are given in Section 3. Section 4 is devoted to linear stability analysis of the stationary and moving fronts of phase transition. In Section 5 we consider dynamics of nonlinear localized disturbances of the moving fronts. In Section 6 we present our conclusion and discussion.

2. Formulation

Let the high permeability water horizon with the water pressure P_0, bounded from below by the plane $z = 0$, be located over the ceiling $z = L$ (the z-axis is directed downwards). The rock in a layer $0 < z < L$ has a low permeability and at the surface $z = L$ it is streamlined by the air of humidity ν_a which is smaller than the humidity of saturation, i.e. the partial pressure in the air is smaller than the pressure of saturation of the vapor in the air at a given value of temperature T. In this case the low permeability porous media $0 < z < L$ contains the water layer $0 < z < h$ and the layer $h < z < L$, saturated by a blend of the air and the water vapor (Fig. 1). The horizontal coordinate varying from $-\infty$ to ∞ is denoted by x.

We assume that there exists the front where the evaporation occurs located between the water saturated domain and the domain containing the homogeneous blend of the air and the vapor. Then, in the domain filled in by the liquid phase the inflow of the water from the high permeability horizon takes place towards the front of evaporation. The vapor arising at the front diffuses through the air-vapor domain in the direction of free surface $z = L$ (the ceiling of the underground construction) streamlined by the air. The vapor diffusion occurs in the case when the partial pressure of the vapor in the neighborhood of the evaporation front is greater than the partial pressure at the free surface $z = L$.

Assuming the water to be incompressible we get the continuity equation and Darcy's law constituting the governing equations in the water saturated domain

$$\operatorname{div} \mathbf{w}_w = 0, \qquad m\mathbf{w}_w = -\frac{k}{\mu_w}\operatorname{grad}(P - \rho_w g z). \tag{2.1}$$

The governing equations in the domain saturated by the air-vapor blend represent the equation of the vapor diffusion and the Clapeyron equations for gases:

$$\frac{\partial \rho_v}{\partial t} = \operatorname{div}(D \operatorname{grad}\rho_v), \quad P_v = \rho_v R_v T, \quad P_a = \rho_a R_a T. \tag{2.2}$$

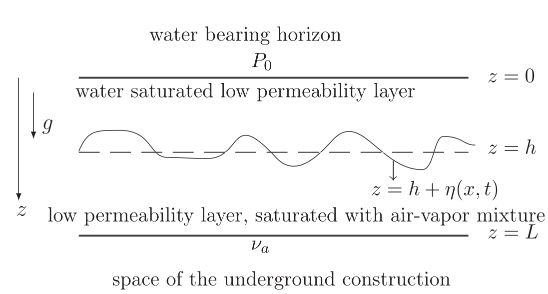

Fig. 1. Schematic of the system considered; see the text for explanations.