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Three dimensional radiative heat transfer model for the evaluation of the anisotropic effective conductivity of fibrous materials

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ABSTRACT

The effective radiative conductivity of fibrous material is an important part of the evaluation of the thermal performance of fibrous insulators. To better evaluate this material property, a three-dimensional direct simulation model which calculates the effective radiative conductivity of fibrous material is proposed. The simplified model assumes that the fibers are in a cylindrical shape and does not require identically-sized fibers or a symmetric configuration. Using a geometry with properties resembling those of a fibrous insulator, a numerical calculation of the geometric configuration factor is carried out. The results show the dependency of thermal conductivity on temperature as well as the orientation of the fibers. The calculated conductivity values are also used in the continuum heat equation, and the results are compared to the ones obtained using the direct simulation approach, showing a good agreement.

1. Introduction

Fibrous materials, such as Phenolic Impregnated Carbon Ablator (PICA) [1], are part of a new generation of light weight insulators used on reentry vehicles for Thermal Protection Systems (TPS) [2–7]. Heat transfer through fibrous materials has been investigated in numerous studies [8–11], and new models have been developed to estimate the TPS material response to high-enthalpy environments [12–14]. In thermal response models, radiation and conduction are two major processes that must be taken into account. Both of these phenomenon require a detailed knowledge of the micro-scale geometry of the material. For radiative transfer, the upscaling task is complex, since the process depends on the specific orientation of individual fibers and their view factors. Different models for radiation and combined radiation–conduction energy transfer in an absorbing and scattering fibrous medium have been previously developed [15–17]. However, these models are limited to materials where the fibers have ordered geometrical features and are oriented parallel to planar boundaries. Hence, developing new models with the ability to calculate the material thermal properties using real geometrical features of the fibers is a significant improvement.

In order to investigate the anisotropic behavior of the fiber preforms, a Direct Simulation (DS) for a two-dimensional fibrous medium has been proposed by van Eekelen and Lachaud [18]. Based on the hypothesis of Rosseland [19], they model the thermal radiation process in a fiber preform medium made of randomly positioned but parallel and identically-sized fibers. Although they show that there is a temperature dependence of the radiative conductivity, the connection to angular dependence was not taken into account. However, it has been shown that light-weight fibrous insulator such as PICA have a preferred orientation of the fibers and, therefore, show anisotropic behavior [20–24].

In the present paper, a robust DS model that covers both the angular and temperature dependency of the radiative conductivity is proposed. The model is used to determine the values for the effective radiative conductivity of an anisotropic fibrous material. The conductivity values are then tested using the DS model combined with an approximate thermal propagation method and are compared to results obtained using a macro-scale volume-averaged heat transfer continuum analysis.

2. Model and test-geometry

2.1. Fiducial volume simulation

While X-ray micro-tomography measurements of PICA [20] show that the overall geometry of the actual fibers is not perfectly

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Nomenclature

Symbols

A	total surface area of a fiber [m^2]
d	distance between cylinders [m]
dA	differential area [m^2]
h	length [m]
K	geometric factor [m]
M	number of cylindrical fibers
m	mass [kg]
N	number of surfaces within enclosure
q	heat flux [$W\,m^{-2}$]
R	rotation matrix
r	radius [m]
S	distance between two differential elements [m]
T	absolute temperature [K]
t	time [s]
V	total volume of a fiber [m^3]
x, y, z	space coordinates [m]
x_0, y_0, z_0	translation matrix elements [m]
c_p	specific heat capacity [$J\,kg^{-1}\,K^{-1}$]

Greek symbols

α, β, γ	Euler angles
δ	Kronecker delta
ϵ	emissivity
κ	radiative conductivity [$W\,m^{-1}\,K^{-1}$]
$\mathfrak{T}r$	transform operator
ϕ	porosity
ρ	effective density [$kg\,m^{-3}$]
ρ_c	carbon fiber density [$kg\,m^{-3}$]
σ	Stefan–Boltzmann constant [$W\,m^{-2}\,K^{-4}$]
τ	iteration counters
ξ	maximum bias angle
ζ	small error due to grid resolution

Subscripts

j, k	indices denoting individual fibers and enclosure internal boundary surfaces
m, n	indices denoting matrix elements
inc	incoming
out	outgoing

Vectors

$\mathbf{p}', \mathbf{p}''$	surface element position of source and target [m]
$\mathbf{p}_1, \mathbf{p}_2$	bottom and top cap center position [m]
$\mathbf{u}$	coordinates on the cylinder
$\mathbf{u}'$	line equation
$\mathbf{v}, \mathbf{v}'$	direction vectors
$\mathbf{x}, \mathbf{x}'$	space coordinates vector [m]

cylindrical, the high porosity of the material makes the cylindrical geometry appropriate for validation studies. As illustrated schematically in Fig. 1, the measurement volume, or fiducial volume, used in the present analysis is assumed to be a cubic enclosure, 1.0 mm (x) $\times$ 1.0 mm (y) $\times$ 1.0 mm (z), which contains the arbitrarily sized, positioned, and oriented cylinders.

The cylindrical fibers are generated in two steps. First, a set of arbitrary values for the radius and length, $\{r_k\}_{k=1}^{M}$ and $\{h_k\}_{k=1}^{M}$, are assigned to M cylinders. In Cartesian coordinates, if the cylinders are generally oriented along the z-axis $(0,0,1)$, then the centers of the bottom and top cap of the kth fiber are $\mathbf{p}_{1,k} = (0, 0, -h_k/2)$ and $\mathbf{p}_{2,k} = (0, 0, +h_k/2)$, respectively. Secondly, the fibers within the 3D geometry (fiducial volume) are rotated and translated using a linear transformation mapping, $\mathfrak{T}r(\mathbf{x}) = \mathbf{x}'$. Rotation and translation are accomplished by using a pair of operators, called the rotation and translation operators. These operators are combined to form the transform operator $\mathfrak{T}r$. The transform operator modifies the observation point $\mathbf{x} = (x, y, z)$ as follows:

$$\mathbf{x}' = \mathfrak{T}r(\mathbf{x}) = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

where R_{mn} and (x_0, y_0, z_0) are rotation and translation matrix elements, respectively. The rotation operator R is an extrinsic rotation defined as three matrix multiplications $R_{\gamma(z)}R_{\beta(y)}R_{\alpha(x)}$, where α, β, γ are random Euler angles about the x-, y-, z-axes, respectively. A transform operator is assigned for each cylinder. Therefore, the main axis, top, and bottom cap of the kth cylinder are respectively transformed to:

$$\mathbf{v}_k = (R_{13}, R_{23}, R_{33})_k,$$

$$\mathbf{p}_{1,k} = \left(x_0 - \frac{h}{2}R_{13}, y_0 - \frac{h}{2}R_{23}, z_0 - \frac{h}{2}R_{33}\right)_k,$$

$$\mathbf{p}_{2,k} = \left(x_0 + \frac{h}{2}R_{13}, y_0 + \frac{h}{2}R_{23}, z_0 + \frac{h}{2}R_{33}\right)_k.$$

The geometrical properties of the fiber used in the model have a radius of $10.0 \leqslant r \leqslant 20.0$ µm and a length of $300.0 \leqslant h \leqslant 600.0$ µm. The non-overlapping fibers (verified using the separating axes method [25]) with random sizes and orientations are positioned irregularly within the fiducial volume until the desired fiber volume fraction of $\simeq 0.2$ is obtained. The fibers have an azimuthal direction oriented between $-15.0^\circ < \xi < 15.0^\circ$ with respect to the xy-plane. This preferred orientation is expected to reduce the thermal conductivity in z-direction. The geometrical layout of the cylindrical fibers used in the present analysis are presented in Fig. 2.

The boundary surfaces and the fibers are referenced using the following set of integer as subscripts:

$$\underbrace{0, 1, 2, 3, 4, 5,}_{\text{Internal boundary surfaces of the enclosure}} \quad \underbrace{6, 7, \ldots, N-1,}_{\text{Surface of the fibers}} \quad N = 6 + M.$$

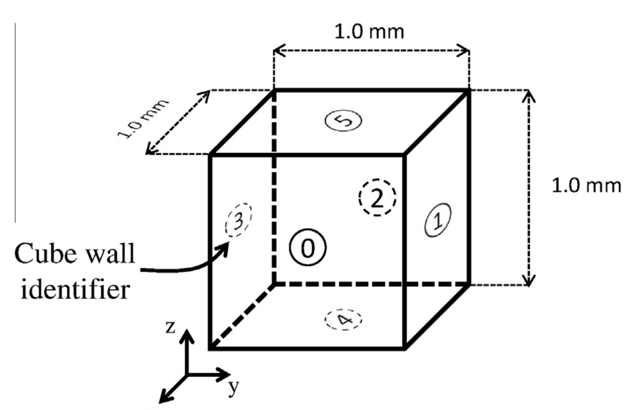

Fig. 1. Schematic figure of the specific experimental geometry employed in the model. The fiducial volume consists of a cubic enclosure whose boundary walls are composed of the same material as the inside fibers. The integers are used to identify the boundary surfaces.