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A filter based solution for inverse heat conduction problems in multi-layer mediums

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ABSTRACT

This paper presents a solution for the inverse heat conduction problem (IHCP) in a multi-layer medium based on solutions from individual layers separately. The approach allows for inclusion of known contact resistances between the layers. The temperature histories are assumed known at two points on the inner layer and the heat transfer rate at the far end of the outer layer is the desired unknown parameter. A step-by-step solution is proposed for solving this problem based on minimization of the sum-of-squared errors between the computed and known temperature values and using Tikhonov regularization for stabilizing the solution. A Tikhonov digital filter solution is developed which allows near real-time heat transfer estimation in multi-layer application. The proposed method is tested via numerical experiments using exact solutions and ANSYS to generate synthetic data.

1. Introduction

The inverse heat conduction problem (IHCP) is defined as the problem of estimating unknown surface conditions (temperature or heat flux) using internal temperature measurements. This problem arises in several industrial applications such as thermal manufacturing processes. The IHCP is an ill-posed problem due to the lack of continuous dependence of the solution on the data. In other words, an error in input data will result in a significant output error. Therefore, an appropriate regularization method needs to be applied to convert the ill-posed problem to a nearby well-posed problem which can be solved. Several techniques have been proposed and applied for solving IHCPs which can be found in references namely Beck [1], Alifanov [2], Ozisik and Orlande [3] and Murio [4]. Some of these methods include the least-square method with regularization, the sequential function specification, conjugate gradient method and numerical approaches [1].

Conduction through multi-layer mediums has been discussed in several references. Ozisik [5] discussed conduction in one dimensional composite media using different approaches including orthogonal expansions, Green's functions and Laplace transform. The transient response of one-dimensional multilayered composite conducting slabs to sudden variations of the temperature of the surrounding fluid is studied by de Monte [6]. Lu et al. [7] developed an analytical method for solving multi-layer heat conduction problems using Laplace transform and separation of variables. They show that the result from their proposed closed form solution is in good agreement with numerical techniques. Haji-Sheikh and Beck [8] studied the temperature field in multi-dimensional, multi-layer bodies for the boundary conditions of the first, second and third kind. A solution for transient heat conduction through a one-dimensional three-layer composite slab is presented by Sun and Wichman [9].

Unlike direct problems, the solution of IHCPs for a multi-layer medium is only discussed in a few studies. Al Najem and Ozisik [10] conducted an inverse heat conduction analysis for estimating the surface condition in composite layers based on a splitting-up procedure and nonlinear least-squares technique for the whole time domain. Ruan et al. [11] calculated the unknown boundary cooling condition and contact heat transfer coefficient for solidification of alloys based on the least square method and using Beck's future time method and a regularization technique to stabilize the solution. A study on the design of optimal transient heat conduction experiments on composite orthotropic materials is performed by Taktak et al. [12]. They considered several geometries for both 1-D and 2-D cases. Al-Najem [13] developed a method of analysis for determining surface conditions from the knowledge of the time variations of the temperature at the insulated boundary. He used two segmented polynomials in time for the unknown surface temperature. An inverse solution is then developed over the whole time domain using the splitting-up procedure.

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Nomenclature

$\boldsymbol{f}$	filter coefficients (coupled solution)
$\mathbf{F}_1$	filter matrix (X22 case, layer 1)
$\mathbf{F}_2$	filter matrix (X21 case, layer 2)
$\boldsymbol{g}$	filter coefficients (coupled solution)
G	Green's function
$\mathbf{G}$	filter matrix (X12 case)
k	thermal conductivity, W/m-K
L	thickness of the layer, m
m_f	number of future time steps
m_p	number of past time steps
n	number of time steps (Eq. (37))
q	heat flux, W/m^2
S	sum of squares of the temperature error, K^2
t	time, s
t_d	dimensionless time step
T	temperature, K
x	spatial coordinate, m
x'	dummy integration variable, Eqs. (1) and (13)
$\mathbf{X}$	sensitivity matrix for unknown surface heat flux
y	measured temperature at boundary $x = L$
Y	measured temperature at location $x = x_1$
$\mathbf{Z}$	sensitivity matrix for measured temperature boundary condition at $x = L_2$
Greek/roman	
α	thermal diffusivity (k/C), m^2/s
α_T	Tikhonov regularization parameter
β	eigenvalue
ϕ	step response function for unit heat flux at $x = 0$
η	step response function for unit temperature at $x = L$
τ	integration variable, Eqs. (1) and (13)
Subscripts	
0	surface location or reference value
c	reference value for non-dimensionalization
i	time index
M	current time step
m	eigenvalue index
ref	suitable reference value (arbitrary)
ss	steady state
$i_{\max}$	last time index
X12	Cartesian heat conduction problem with type 1 and type 2 boundary conditions
X21	Cartesian heat conduction problem with type 2 and type 1 boundary conditions
X22	Cartesian heat conduction problem with type 2 and type 2 boundary conditions
Superscript	
~	dimensionless parameter

Recently several research works have been performed to investigate real-time or filter forms for processing temperature data to solve IHCPs. A filter solution based on the idea of training neural networks is studied by Kowsari et al. [14]. Ijaz et al. [15], used a Kalman filter to solve a two-dimensional transient IHCP. Feng et al. [16] used Laplace transforms to relate the measured conditions at one end of a domain to the unknown conditions at the remote surface. Woodbury and Beck [17] studied the structure of the Tikhonov regularization problem and concluded that the method can be interpreted as a sequential filter formulation for continuous processing of data. They show that the computed heat fluxes using the whole domain solution and the filter coefficient solution are virtually the same for the constant-property solutions.

In most of the IHCP studies, the heating condition on the remote boundary is assumed as an insulated surface or cooled with a known heat transfer coefficient, e.g., [15,18,19]. However, in practice such ideal conditions are not easy to attain. Most recently Woodbury et al. [20] developed a filter-based solution to incorporate the temperature measurement history from a second subsurface sensor as a remote boundary condition in an IHCP solution. An example of such a problem in industry is the Directional Flame Thermometer (DFT) which is an equilibrium heat flux sensor that is used to estimate the heat flux using temperature measurements [21].

In the present paper, a solution for the IHCP is proposed for a two-layer medium when the temperature measurement history is given in two interior locations of the inner layer. A step-by-step solution is proposed for solving this problem based on the minimization of the sum of the squared errors between the computed and known values and using Tikhonov regularization (TR) for stabilizing the solution. The resulting algorithm is written in filter form. The filter form solution can be used for near real time heat flux estimation. The proposed solution is then demonstrated through several numerical experiments. The filter solution of the IHCP has a number of advantages including simplicity, continuous operation and application to moderate nonlinearity [22] which makes it an appropriate approach for real time heat flux estimation in industrial applications. It is noteworthy that when the material properties are temperature dependent, the problem is no longer linear. For this type of problem, the filter coefficients for a range of temperatures can be calculated and then, using linear interpolation, the values of filter coefficients can be found for each time step based on the current level of temperature.

2. Problem definition

It should be noted that the method presented in this paper can be applied on a medium with more than two layers as long as there are two temperature measurements available on the inner layer. However, a two layer medium is considered to demonstrate the application of the proposed approach. Basically, an IHCP is solved for each layer, starting from the one with known temperature measurements, and the heat flux is estimated at the interface with the next layer. A schematic of a two-layer slab is shown in Fig. 1. As seen, in Fig. 1, the temperature measurement histories are available in the innermost layer (layer 2) for $x = x_1$ and $x = x_2$ while no specific temperature/heat flux measurement is available on the other layer(s) (here, layer 1). The desired unknown parameter is the heat flux at the remote surface of the outer layer (layer 1, $x = 0$).

3. Solution strategy for the multi-layer IHCP

Two IHCPs are solved separately for heat flux estimation at $x = 0$ and eventually a coupled solution is derived for the two-layer problem. A schematic of the system is given in Fig. 1. The solution is started in the inner layer, where two temperature measurements are available at x_1 and $x_2 = L_1 + L_2$ and q_1 is the unknown heat flux