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On the multiple solutions of boiling fins with heat generation

Rizos N. Krikkis

Institute of Thermal Research, I. Fokianou 16 Str, PO Box 116 35, Athens, Greece

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This paper is dedicated to the memory of the late professors Stratis V. Sotirchos and Panagiotis Razelos.

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ABSTRACT

A numerical bifurcation analysis is carried out in order to determine the solution structure of pin fins subject to multi-boiling heat transfer with internal heat generation. The analysis reveals that a complex and interesting solution structure exists. In contrast to the case without heat generation where up to three solutions exist, a large number of solutions may emerge arranged in a nested series of cusp points and isolas as the internal heat generation rate increases. The increased number of multiplicities imposes a number of both monotonic and non-monotonic temperature distributions along the fin height and reversal of the heat flow is observed at certain operating conditions. The fin profile has a profound effect on the solution by either increasing or decreasing the number of the singular points depending on whether a cylindrical or a trapezoidal profile is utilized. A generalized and smooth boiling heat transfer coefficient has been adopted so the results are not restricted to a particular working fluid.

1. Introduction

Boiling phenomena on extended surfaces have been extensively investigated analytically, numerically and experimentally since high heat duties can be attained and under certain operating conditions all boiling modes may be present under stable steady state. Westwater and co-workers [1,2] were among the first to study the boiling mechanism on pin fins. The authors determined experimentally the heat transfer coefficient and used successfully a one-dimensional model including the effects of radiation for the prediction of the extended surfaces heat duty. Similar theoretical as well as experimental investigations were carried out at the Institute for High Temperatures of the Russian Academy of Sciences by Petukhov, Kovalev and co-workers [3–5], where a method for measuring the local heat transfer coefficient was developed and subsequently the temperature distributions and the heat duties for a cylindrical spine have been measured. Expressing the heat transfer coefficient as a power law function greatly facilitates the computations. Analytical solutions for cylindrical spines have been obtained by Liaw and Yeh [6], while Lin and Lee [7] conducted a linear stability analysis for pin fins under two and three mode boiling using polynomial as well as trigonometric basis eigenfunctions. Numerical bifurcation analysis has been conducted for various pin fin profiles by Krikkis et al. [8]. Similar works have been reported for fins with different geometries such as longitudinal [9,10] and radial fins [11–14].

As it is obvious from the literature review in the preceding paragraph most of the work has been focused on fins without heat generation. Yet extended surfaces with internal heat generations are applicable to the fields of nuclear reactor engineering and scientific instrumentation (hot-wire anemometry). Kovalev and Rybchinskaya [15] examined multi-boiling heat transfer with internal heat generation on the performance of a cylindrical pin fin. Specifically when both mechanisms are present, for a given base heat flux the existence of four different base temperature difference (TD) values are possible. This already suggests that more than three solutions may be expected at certain conditions. Unal, [16,17] considered uniform and non-uniform (temperature dependent) internal heat generation together with a power law heat transfer coefficient to obtain analytical solutions for rectangular longitudinal and cylindrical pin fins. Unique solutions to the problem have been reported. Recently Lu et al. [18] examined the multiple solutions and its stability during boiling on a thin cylinder with internal heat generation from an electrical source.

In the present study a one-dimensional conduction–convection model is employed for the numerical bifurcation analysis of pin fins of variable profile subjected to multi-boiling heat transfer conditions with uniform internal heat generation. The code developed and validated during previous works [8,10,13], is being extended and two different algorithms are employed for the

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E-mail address: rkrik@uth.gr

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Nomenclature

a_j	constants in heat transfer coefficient, Eq. (13) (–)
A	(πY^2) surface area (m^2)
Bi	($2h_{ref}r_b/k$) Biot number (–)
D	heat duty function defined by Eq. (12) (–)
h	boiling heat transfer coefficient (W/(m^2 K))
G	[$q_g r_b/2(h\Delta T)_{ref}$] generation number, Eq. (3) (–)
k	fin thermal conductivity (W/(m K))
L	fin height (m)
n	profile exponent in Eq. (9) (–)
P	($2\pi Y$) perimetry (m)
q_f	fin heat loss per unit length (W)
Q	dimensionless fin heat dissipation (–)
q_g	internal heat generation rate per unit volume (W/m^3)
Q_g	(yG) reduced internal heat generation rate (–)
Q_r	reduced boiling heat flux (–)
r_b	base radius (m)
T	temperature (K)
X	distance along fin (m)
x	(X/L) dimensionless distance along fin (–)
y	(Y/r_b) dimensionless profile radius (–)
Y	profile radius (m)
u	conduction–convection parameter (–)

Greek symbols

ΔQ	($Q_r - Q_g$) reduced heat flux difference (–)
ΔT	($T - T_\infty$) temperature difference (K)
Θ	[$(T - T_\infty)/(T_{ref} - T_\infty)$] dimensionless fin temperature (–)
λ	(r_e/r_b) tip to base fin radius ratio (–)

Subscripts

b	fin base
e	fin tip
r	reduced value (i.e. $h_r = h/h_{ref}$)
ref	reference value
u	reference to uniform solutions $\Theta_e = \Theta_b$
∞	ambient boiling liquid

Superscripts

($'$)	derivative with respect to x

Abbreviations

CCP	conduction–convection parameter
TD	temperature difference
TP	turning point(s)

numerical solution of the governing equations. The multiplicity structure is obtained in order to determine the different types of bifurcation diagrams, and the maximum number of the singular points which describe the dependence of a state variable of the system (for instance the fin temperature or the heat dissipation) on a design (conduction–convection parameter, generation number) or operational parameter (base temperature).

2. Analysis

Consider a uniform density pin fin depicted schematically in Fig. 1 with profile $Y = Y(X)$ and constant thermal conductivity k. The fin has base radius r_b, tip radius r_e and height L. The base of the fin is maintained at constant temperature T_b with the surrounding liquid at boiling temperature T_∞. Following Krikkis et al. [8,10] and assuming a uniform internal heat generation rate per unit volume q_g, the following differential equation for the fin temperature is obtained:

$$\frac{d^2\Theta}{dx^2} = \frac{u^2\Delta Q - 2y'\Theta'}{y}, \quad 0 \leqslant x \leqslant 1, \tag{1}$$

where $\Theta = (T - T_\infty)/(T_{ref} - T_\infty)$ is the dimensionless fin temperature,

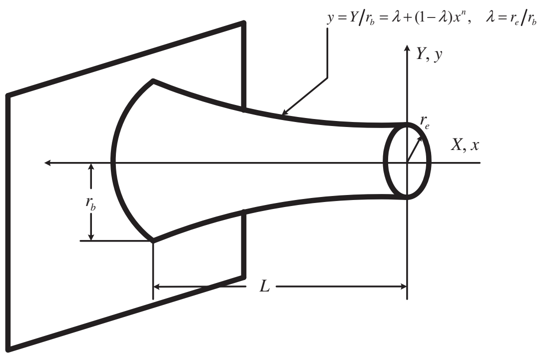

Fig. 1. Pin fin geometry.

$$\Delta Q = Q_r - Q_g = h_r\Theta - yG \tag{2}$$

is the reduced heat flux difference between the boiling and the internal heat generation processes,

$$G = q_g r_b/2(h\Delta T)_{ref} \tag{3}$$

is the dimensionless generation number and

$$h_r = \frac{h(\Theta\Delta T_{ref})}{h_{ref}} \tag{4}$$

is the reduced heat transfer coefficient while Θ' represents derivative with respect to x, the dimensionless distance along the fin Fig. 1. Furthermore, the dimensionless parameters that describe the problem is the conduction–convection parameter (CCP),

$$u^2 = \frac{2h_{ref}L^2}{kr_b} = \frac{L^2}{r_b^2}\mathrm{Bi}, \tag{5}$$

which interrelates the geometrical (aspect ratio L/r_b) and the thermal parameters of fin through the Biot number,

$$\mathrm{Bi} = 2h_{ref}r_b/k, \tag{6}$$

which is a measure of the resistance to conduction within the fin to the convective resistance across the boiling liquid–fin interface. In the above equation, $h_{ref} = h(\Delta T_{ref})$ denotes the heat transfer coefficient at a reference temperature difference ΔT_{ref}. The dimensionless boundary conditions are:

$$\Theta(1) = \Theta_b, \tag{7}$$

$$\Theta'(0) = \left(\frac{h_e L}{k}\right)\Theta(0) = \left(\frac{h_e}{2h_{ref}}\right)u\mathrm{Bi}^{1/2}\Theta_e. \tag{8}$$

The equation that describes the profile of the fin is

$$y(x) = Y/r_b = \lambda + (1-\lambda)x^n, \tag{9}$$

where $\lambda = 1$ and $n = 0$ corresponds to the cylindrical profile, whereas for the trapezoidal profile both values $\lambda < 1$ and $\lambda > 1$ for the tapper ratio may be used. It can be seen from Eq. (1) that the dimensionless temperature Θ and its derivative Θ' have the form:

$$\Theta = \Theta(x; u, G, \Theta_b, \lambda, \mathrm{Bi}), \quad \Theta' = \Theta'(x; u, G, \Theta_b, \lambda, \mathrm{Bi}). \tag{10a, b}$$