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Entropy generation in MHD porous channel with hydrodynamic slip and convective boundary conditions

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ABSTRACT

Combined effects of hydrodynamic slip, magnetic field, suction/injection and convective boundary conditions on the global entropy generation in steady flow of an incompressible electrically conducting fluid through a channel with permeable plates are studied. Analytical solutions of the governing equations are obtained in closed form. Particularly, using thermal boundary conditions of the third kind, the temperature field is derived analytically. Also, the influences of the governing parameters on global entropy generation are discussed in detail and depicted graphically. The analysis of our results indicates that entropy generation minimization can be achieved by appropriate combination of the geometrical and physical parameters of the system. It is possible to determine optimum values of Hartmann number, Biot number and Prandtl numbers which lead to a minimum global entropy generation rate. The effects of slip flow on the optimum values of some other parameters are also explored.

1. Introduction

Efficient utilization of energy is the mean objective in the design of thermal devices. This can be achieved by minimizing entropy generation in processes. Therefore, in energy optimization problems and design of many heat removal engineering devices, it is necessary to minimize the entropy generation or destruction of available work due to heat transfer, viscous friction and electric conduction as a function of the design variables selected for the optimization analysis. The search for conditions that lead to minimization of entropy generation in a given process or device under various flow configurations has been the task of several investigations reported in the literature [1–14]. In these works, the results showed that the geometrical and physical parameters of the system might be chosen in order to minimize entropy generation in the system. In particular, Aziz and Khan [3] studied the entropy generation for steady conduction in a slab with temperature-dependent volumetric internal heat generation. They found that the total entropy generation in the slab can be minimized with a suitable choice of the cooling parameters. In another paper [4], they analyzed the classical and minimum entropy generation for steady state conduction with temperature dependent thermal conductivity for steady conduction in a plane wall, a hollow cylinder and a hollow sphere. The results indicated that both the classical and minimum entropy generation rates, for each geometry, to be strong functions of thermal asymmetry and the thermal conductivity variation parameter. Mahian et al. [5] studied the effects of nanofluids on entropy generation between two cylinders in the presence of a magnetic field. Torabi et al. [6,7] examined the effects of convective-radiative boundary conditions on entropy generation rate in an asymmetrically cooled hollow cylinder and in cooled homogenous slabs, respectively. Ibáñez et al. [8,10,13] investigated the global entropy generation in microchannels by considering the conjugate heat transfer problem in the fluid and solid walls. They examined ordinary microfluidics [8,10] and magnetohydrodynamic microfluidics [13]. In these papers, special attention was given to the effects of the slip velocity [8,10] and wall thickness [13] on the entropy generation.

In this context, the analysis of fluid flow and heat transfer in porous channels in the presence of an electromagnetic field became an attractive area of research due to its numerous engineering and industrial applications such as in heat exchangers, cooling of electronic devices and a variety of devices related to the electromagnetic processing of materials [15]. In particular, some recent works have addressed the analysis of entropy generation in porous channels considering mainly the effects of viscous and thermal irreversibilities [16–21]. Mahmud and Fraser [16] presented a numerical solution for the flow, thermal and entropy generation characteristic inside a porous channel with viscous dissipation. Makinde and Eegunjobi [17] studied the effects of convective heating on entropy generation rate in a channel with

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permeable walls. Makinde and Osalusi [18] analyzed the entropy generation rate in a liquid film falling along an inclined porous heated plate. Eegunjobi and Makinde [19] studied the combined effect of buoyancy force and Navier slip on local entropy generation in a vertical porous channel with walls at uniform temperature. In another task [20], they studied the effects of Navier slip on local entropy generation in a porous channel with suction/injection, once more with walls at uniform temperature. Chinyoka and Makinde [21] presented the combined effects of Navier slip, convective cooling, variable viscosity, and suction/injection on the local entropy generation rate in an unsteady flow of an incompressible viscous fluid flowing through a channel with permeable walls. Makinde and Osalusi [22] investigated the entropy generation in a liquid film falling along an incline porous heated plate.

On the other hand, the dissipative processes that arise in a porous channel flow subject to electromagnetic interactions have been analyzed in [23–31]. Here, in addition to irreversibilities produced by fluid friction and heat and mass transfer, Joule dissipation generated by electric currents circulating in the conducting fluid has to be considered. Rashidi et al. [23] studied the entropy generation in an incompressible nanofluid flowing over a rotating porous disk in the presence of a magnetic field. Butt and Ali [24] considered the effects of entropy generation in magnetohydrodynamic (MHD) flow over a permeable stretching sheet embedded in a porous medium in the presence of viscous dissipation. Analytical solutions of the momentum and the energy equations were obtained in terms of Kummer's function using suitable transformations, also the entropy generation number Ns and the Bejan number Be were calculated. Yazdi et al. [25] examined embedded open parallel microchannels within a micropatterned permeable surface for reducing entropy generation in MHD fluid flow in microscale systems. A local similarity solution for the transformed governing equations was obtained and the entropy generation numbers, as well as the Bejan number, were investigated. Tasnim et al. [26] studied the entropy generation in a porous channel with hydromagnetic effects. Makinde and Eegunjobi [27] analyzed the inherent irreversibility in a variable viscosity MHD generalized Couette flow with permeable walls. Eegunjobi and Makinde [28] examined the local entropy generation in a variable viscosity MHD channel flow with permeable walls and convective heating. Makinde and Chinyoka [29] discussed the numerical investigation of buoyancy effects on MHD unsteady flow in a vertical porous channel. Das and Jana [30] examined the effects of magnetic field and suction/injection on the entropy generation in MHD porous channel flow under constant pressure gradient. In a subsequent paper, Das and Jana [31] included the effects of Navier slip on the local entropy generation rate in an MHD flow through a porous channel under a constant pressure gradient and uniform temperature at the walls. Although the entropy generation rate was calculated in these MHD porous channels, the combined effects of hydrodynamic slip and convective boundary conditions were not considered and the analysis mainly focused on the local entropy generation.

In the present contribution, the local entropy generation rate is integrated in the whole region of analysis so that the finite dimensions of the device are considered to obtain the global entropy generation rate. This latter quantity is discussed in detail and investigated in MHD porous channel considering hydrodynamic slip, suction/injection Reynolds number and thermal boundary conditions of the third kind at the walls. A parametric study is carried out to see how the mean parameters of the problem affect the global entropy generation. In fact, including convective boundary conditions in the evaluation of temperature field enables us to optimize not only the magnetic field and the flow parameters but also the rate of convective heat transfer at the walls. Such a problem may be important for the design of cooling and heat transfer devices but has to the best of my knowledge not been studied previously. It is noted that the present solution extends the work of Das and Jana [31] to include the combined effects of slip flow, magnetic field, suction/injection and convective heat transfer on the global entropy generation using an analytical solution for the velocity and temperature fields.

The main contributions are: (a) The combined effects of hydrodynamic slip, magnetic field and suction/injection Reynolds number on the global entropy generation rate are studied under convective boundary conditions. (b) For certain suitable combination of the geometrical and physical parameters of the system the global entropy generation rate is minimized. (c) In particular, optimal values of Hartmann, Biot, Reynolds and Prandtl numbers which lead to a minimum global entropy generation rate are obtained. (d) The effects of slip flow on the optimum values of Hartmann, Biot, Reynolds and Prandtl numbers are analyzed.

In the following sections, the problem is formulated, analyzed, analytically solved and discussed. Section 2 consists of the transport problem analysis which contains the momentum and energy balance equations and their solutions. Section 3 contains the determination of the local and global entropy generation, and graphical representation of the relevant results and their discussion. Concluding remarks follow in Section 4.

2. The model fluid and the governing equations

We consider the steady fully developed flow of an incompressible viscous fluid through a channel with two horizontal parallel porous plates separated by a distance $2a$ in the presence of a constant longitudinal pressure gradient dp/dx' and under a uniform transverse magnetic field B_0. The upper plate is located at $y' = a$ and the lower plate is at $y' = -a$, y', denoting the transversal coordinate. It is assumed that the parallel plates are infinite so that the velocity and temperature profiles are fully developed, also the fluid is injected uniformly into the channel at the lower plate and fluid suction occurs at the upper plate. The channel lower plate exchanges heat by convection with a hot fluid with temperature T_h while the upper plate is in contact with the ambient. For the solution of the momentum balance equation we assume that the velocity satisfies the slip condition at the plates. In turn, the heat transfer equation is solved using boundary conditions of the third kind. A schematic view of the MHD porous channel is shown in Fig. 1.

2.1. Velocity and temperature fields

Given the previous assumptions the momentum equation is

$$v_0 \frac{du'}{dy'} = -\frac{1}{\rho}\frac{dp}{dx'} + \frac{\eta}{\rho}\frac{d^2u'}{dy'^2} - \frac{\sigma B_0^2}{\rho}u'. \tag{1}$$

Let us assume that the surface roughness of each plate is in general different. Then, the slip lengths, although taken to be constant, do not have the same value on both plates. Therefore, Eq. (1) must satisfy the boundary conditions

$$u' + \alpha_1' \frac{du'}{dy'} = 0, \quad \text{at} \quad y' = a, \tag{2}$$

$$u' - \alpha_2' \frac{du'}{dy'} = 0, \quad \text{at} \quad y' = -a, \tag{3}$$

where v_0 is the uniform suction/injection velocity at the channel plates and a is the half separation between the planar plates. Here, η is the dynamic viscosity, σ is the electrical conductivity of the fluid, ρ is the density of the fluid, while α_1' and α_2' are the slip lengths of the upper and lower plates, respectively, which in general, are assumed to be different.