International Journal of Heat and Mass Transfer 80 (2015) 614–625

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Magnetoconvection of a micropolar fluid in a vertical channel

A. Borrelli, G. Giantesio *, M.C. Patria

[a] Dipartimento di Matematica e Informatica, Università di Ferrara, via Machiavelli 35, 44121 Ferrara, Italy

ARTICLE INFO

Article history:
Received 2 July 2014
Received in revised form 16 September 2014
Accepted 16 September 2014
Available online 16 October 2014

Keywords:
Micropolar fluids
MHD fully developed flow
Mixed convection
Natural convection
Boussinesq approximation
Vertical channel

ABSTRACT

This work examines the effects of the magnetic field and of the temperature on the steady mixed convection in the fully developed flow of a micropolar fluid filling a vertical channel under the Oberbeck–Boussinesq approximation. The two boundaries are considered as isothermal and kept either at different or at equal temperatures. The velocity, the microrotation, the temperature and the induced magnetic field are analytically obtained. A selected set of pictures illustrating the influence of various parameters involved in the problem (the coupling parameter, the micropolar parameter, the Hartmann number and the buoyancy coefficient) is presented and discussed. Moreover, the results obtained for the micropolar flow are compared with the corresponding for the Newtonian fluid.

1. Introduction

The recent industrial processes are characterized by the use of new materials which cannot be described by Newtonian fluids. Due to this reason, many non-Newtonian models have been proposed. Among these models, the micropolar fluids have been introduced by Eringen [1] in order to take into account the effects of local structure and micro-motions of the fluid particles which cannot be described by the classical models. The incompressible micropolar fluids represent liquids consisting of rigid, randomly oriented spherical particles suspended in a viscous medium, where the deformation of fluid particles is ignored. The related mathematical model is based on the introduction of a new vector field (the microrotation) which describes the total angular velocity field of the particles rotation. Hence, one new equation is added representing the balance law of local angular momentum.

There are many examples of flows of micropolar fluids that are relevant for practical applications as flows of biological fluids in thin vessels, polymeric suspensions, liquid crystals, slurries, colloidal fluids, exotic lubricants, etc. Extensive reviews of the theory and its applications can be found in [2,3].

The purpose of the present paper is to study the influence of an external uniform magnetic field on the mixed convection in the fully developed flow of a micropolar fluid filling a vertical channel under the Oberbeck–Boussinesq approximation. A systematic and rigorous derivation of this approximation is provided in [4].

Convection flow of an electrically conducting fluid in a channel under the effect of a transverse magnetic field has a relevant technical significance because of its many industrial applications such as geothermal reservoirs, cooling of nuclear reactors, electric transmission cables, thermal insulation and petroleum reservoirs, to name a few.

In our study we solve the problem of the mixed convection of a Boussinesquian electrically conducting micropolar fluid which steadily flows in a vertical channel under the action of a uniform magnetic field applied normal to the direction of the velocity. The walls are maintained at constant temperatures T_1 and T_2 ($T_1 \leqslant T_2$).

The first paper on the fully developed free convection of a micropolar fluid in a vertical channel is [5]; this work has been generalized in [6] in order to consider also the mass transfer. In [7–9] mixed convection flow with symmetric and asymmetric heating is examined. To the best of our knowledge, few results are known concerning the influence of an external magnetic field on the convective flow of a micropolar fluid in a vertical channel [10,11], while in recent years the same situation in a double channel has been studied in [12]. However, in most of the previous papers, a restrictive condition on the material parameters has been imposed following the work of Ahmadi [13]. We point out that in our research we have not required any condition so that two

* Corresponding author.
E-mail addresses: brs@unife.it (A. Borrelli), gntgli@unife.it (G. Giantesio), pat@unife.it (M.C. Patria).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.09.031
0017-9310/

Nomenclature

b	thermal diffusivity
C	constant such that $P = -Cx_1 + p_0$
$c_0,\ c_d,\ c_a$	angular viscosity coefficients
$2d$	channel width
$\mathbf{E}$	electric field
$\mathbf{g} = -g\mathbf{e}_1$	gravity acceleration
Gr	Grashof number defined by $(6)_9$
$\mathbf{H}$	total magnetic field
$h(y)$	dimensionless function describing the induced magnetic field defined by $(6)_{15}$
$H_0\mathbf{e}_2$	external uniform magnetic field ($H_0 > 0$)
$H_1(x_2)$	induced magnetic field component in the x_1-direction
I	microinertia coefficient
k	fluid thermal conductivity
$k_{1,2}$	heat transfer coefficients evaluated at $\Pi_{1,2}$
l	characteristic length defined by $(6)_2$
L	dimensionless constant defined by $(6)_3$
M^2	Hartmann number defined by $(6)_5$
M_p^2	micropolar parameter defined by $(6)_4$
N^2	coupling number defined by $(6)_1$ ($0 < N^2 < 1$)
Nu	Nusselt number
p	pressure
$P = p + \mu_e \frac{H_1^2}{2} + \rho_0 g x_1$	difference between the hydromagnetic pressure and the hydrostatic pressure
p_0	arbitrary constant
$\mathbf{q}$	heat flux vector
Re	Reynolds number defined by $(6)_8$
$T = T(x_2)$	temperature
T_0	reference temperature
$T_1,\ T_2$	uniform temperatures ($T_2 \geqslant T_1$)
$\mathbf{v}$	velocity field
$v(y)$	dimensionless function describing the velocity defined by $(6)_{12}$
V_0	characteristic velocity defined by $(6)_6$
$v_1(x_2)$	velocity component in the x_1-direction
$\mathbf{w}$	microrotation field
$w(y)$	dimensionless function describing the microrotation defined by $(6)_{13}$
$w_3(x_2)$	microrotation component in the x_3-direction
y	dimensionless transverse coordinate defined by $(6)_{11}$
Greek symbols	
$\alpha,\ \beta,\ \gamma$	dimensionless constants defined by (11)
α_T	thermal expansion coefficient
η_e	electrical permittivity $\left(\eta_e = \frac{1}{\mu_e \sigma_e}\right)$
$\vartheta(y)$	dimensionless temperature defined by $(6)_{14}$
λ	buoyancy coefficient defined by $(6)_{10}$
μ	Newtonian viscosity coefficient ($\mu > 0$)
μ_e	magnetic permeability
μ_r	dynamic microrotation viscosity coefficient
ν_0	constant defined by $(6)_7$
ρ_0	mass density at the temperature T_0
σ_e	electrical conductivity
$\tau_{1,2}$	skin friction at the plates $\Pi_{1,2}$
$\tau_{p1,2}$	skin couple friction at the plates $\Pi_{1,2}$

material parameters describe the micropolar nature of the fluid, instead of one as in the simplified Ahmadi's approach.

In our paper, as it is usual in the Oberbeck–Boussinesq approximation [14], we neglect the dissipation terms in the energy equation, so that we can obtain the explicit solution of the problem which takes into account the induced magnetic field. We point out that the induced magnetic field is neglected in most of the works concerning the convective flow in a vertical channel, also in the simpler case of a Newtonian fluid.

The paper is organized in this way:

In Section 2 we formulate the problem from the physical point of view. In order to determine the analytical solution, we have to distinguish three cases which are related to the strength of the external uniform magnetic field.

Section 3 is devoted to integrate the boundary value problem which describes the motion in the three cases.

In Section 4 we make some comments about the flow and we give the solution when the heating is symmetric, in the case of natural convection, in the absence of magnetic field and in the same problems for the Newtonian fluid.

The trend of the solution is plotted in Section 5 in order to show the influence of the relevant parameters on the flow. The behavior of the micropolar flow differs highly from the Newtonian one as the coupling number increases and the micropolar parameter M_p decreases. For suitable values of the buoyancy parameter λ, the reverse flow occurs near the coldest (hottest) wall if $\lambda > 0$ ($\lambda < 0$). The presence of the external magnetic field tends to prevent the occurrence of the reverse flow. If the buoyancy parameter vanishes (symmetric heating), then the phenomenon of the reverse flow does not appear.

Section 6 summarizes the results.

2. Formulation of the problem

Let us consider a Boussinesquian, electrically conducting micropolar fluid filling the region $\mathcal{S}$ between two infinite rigid, fixed, non-electrically conducting vertical plates $\Pi_1,\ \Pi_2$ separated by a distance $2d$ (Fig. 1).

We assume the regions outside the plane to be a vacuum (free space). The coordinate axes are fixed in order to have

$$\mathcal{S} = \{(x_1,x_2,x_3) \in \mathbb{R}^3 : (x_1,x_3) \in \mathbb{R}^2, \quad x_2 \in (-d,d)\},$$
$$\Pi_i = \{(x_1,x_2,x_3) \in \mathbb{R}^3 : (x_1,x_3) \in \mathbb{R}^2, \quad x_2 = (-1)^i d\}, \quad i = 1,2 \tag{1}$$

and x_1-axis is vertical upward.

Our aim is to study the steady mixed convection in the fully developed flow of the fluid under the action of an external uniform magnetic field $H_0\mathbf{e}_2$ normal to planes $\Pi_{1,2}$ ($H_0 > 0$).

This flow in the absence of external mechanical body forces, body couples and free electric charges under the Oberbeck–Boussinesq approximation is governed by [3,1,2]

$$\begin{aligned}
&\rho_0 \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + (\mu+\mu_r)\triangle\mathbf{v} + 2\mu_r(\nabla\times\mathbf{w}) \\
&\qquad + \mu_e(\nabla\times\mathbf{H})\times\mathbf{H} + \rho_0[1-\alpha_T(T-T_0)]\mathbf{g}, \\
&\rho_0 I\mathbf{v}\cdot\nabla\mathbf{w} = (c_a+c_d)\triangle\mathbf{w} + (c_0+c_d-c_a)\nabla(\nabla\cdot\mathbf{w}) \\
&\qquad + 2\mu_r(\nabla\times\mathbf{v} - 2\mathbf{w}), \\
&\nabla\cdot\mathbf{v} = 0, \\
&\eta_e\triangle\mathbf{H} = \nabla\times(\mathbf{H}\times\mathbf{v}), \\
&\nabla\cdot\mathbf{H} = 0, \\
&\nabla T\cdot\mathbf{v} = b\triangle T, \quad \text{in } \mathcal{S}.
\end{aligned} \tag{2}$$