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The extended Graetz problem for a gaseous slip flow in micropipe and parallel-plate microchannel with heating section of finite length: Effects of axial conduction, viscous dissipation and pressure work

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ABSTRACT

The extended Graetz problem for a gaseous slip flow through a micropipe and a parallel-plate microchannel, with an isothermal heating section of finite length, is analytically investigated. The simultaneous effects of the axial heat conduction, viscous dissipation and pressure work are all taken into account and discussed. The solution obtained is based on a powerful method using self-adjoint formalism, resulting from a decomposition of energy equation into a system of the first-order partial differential equations. This solution, which is applicable for finite and semi-infinite heating section, represents an improvement and extension of those obtained in the earlier works, by considering the slip boundary conditions at the fluid–wall interface for the velocity and temperature. This extension has been done by using a new matrix operator of three dimensions and a suitable scalar product between two vectors in the Hilbert space. The analytical results are compared for simplified limiting cases with available analytical and numerical calculations and a good agreement is found. The results of the effects of different dimensionless parameters involved in the problem, namely Péclet, Knudsen, Brinkman numbers and the length of the heating section, on the heat transfer characteristics are illustrated and discussed. Furthermore, some useful correlations of these characteristics are provided for some values of Péclet number. It is shown particularly that for non-zero values of Brinkman number, when the heat flow is established, the sum of the enthalpy and the energy which results from the friction and pressure work is conserved through cross-sections of microchannels, and the heat transfer is mainly governed by the shear work at the wall. Among the most important applications of this analytical solution is its potential to simulate an isothermal hot film sensor of finite size, mounted on the wall of microchannel, which can be serve to measure a heat flux between the gas and wall and hence heat transfer coefficient in the slip flow regime.

1. Introduction

The forced convective heat transfer at the microscale is among of the major themes in Micro-Electro-Mechanical Systems (MEMS) and plays central role in thermal transport phenomena in microfluidic [1]. Microfluidic is referred to devices that have characteristic length scales of the order between 1 μm and 1 mm such as micropipe, parallel-plate microchannel and micro-exchangers.

This study is devoted to the non trivial fundamental problem in convection–diffusion: the Graetz problem extended to microscale in the slip flow regime. Specific consideration is given here to gas flow through a micropipe and a parallel-plate microchannel, with a heating section of finite length, at relatively low Mach number. The analysis of this problem is important, since it is often implicated in industrial processes. Thus, it may find applications in many fields, such as in cooling microelectronic components, conception of the micro-exchangers, chemical engineering, biomedical use and microbiological systems. Moreover, it can be involved in the separation technology of some substances in mass transfer, by analogy with heat transfer.

The basic parameter characterizing the slip flow in microchannels is the Knudsen number $Kn = \lambda/D_h$, which is the ratio of the molecular mean free path λ to the hydraulic diameter D_h. When Kn is considered as the basic parameter, the interval of slip flow regime may be estimated as [1]: $(10^{-3} \leqslant Kn \leqslant 0.1)$ and the gas becomes slightly rarefied. In this case, it can be shown that the flow and the heat transfer phenomena can be safely modeled using continuum conservation equations together with appropriate slip boundary conditions. These boundary conditions are essentially

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Nomenclature

C_p	specific heat at constant pressure J kg^{-1} K^{-1}
F_v	tangential momentum coefficient
F_t	thermal accommodation coefficient
k	thermal conductivity, W m^{-1} K^{-1}
Kn	Knudsen number, $Kn = \lambda/D_h$
Nu	local Nusselt number, Eq. (59)
$\overline{Nu}$	average Nusselt number, Eq. (74), $\overline{Nu} = \bar{h}D_h/k$
P	pressure, Pa
Pe_{Dh}	Péclet number based on the hydraulic diameter, $Pe_{Dh} = 2^{2-p}Pe_H$
Pr	Prandtl number, $\mathrm{Pr} = \nu/\alpha$
q_w	dimensionless heat flux, Eq. (59)
T	temperature, K
T_b	bulk temperature, Eq. (58), K
T_w	wall temperature, K
U_m	mean velocity, m s^{-1}
u_s	slip velocity
x_1	length of the heated zone, m

Greek symbols

α	thermal diffusivity, $\alpha = k/\rho C_p$, m^2 s^{-1}
β_v	parameter, $\beta_v = (2 - F_v)/F_v$
β_t	parameter, $\beta_t = \frac{2-F_t}{F_t}\frac{2\gamma}{\gamma+1}\frac{1}{\mathrm{Pr}}$
γ	ratio of specific heats, $\gamma = C_p/C_v$
λ	mean free path, m
μ	dynamic viscosity, Pa s
Θ	dimensionless temperature, Eq. (12)
Θ_b	dimensionless bulk temperature, $(T_b - T_0)/(T_1 - T_0)$
ρ	fluid density, kg m^{-3}

characterized by discontinuities of the velocity and the temperature fields at the fluid–wall interface due to rarefaction. However, in microscale gas flows with heat transfer for a relatively low velocity (subsonic flow) we encounter many important effects that are not generally considered in macroscale channels. These effects are: rarefaction, axial heat conduction, compressibility, viscous dissipation and pressure work, and may be characterized and scaled respectively by the Knudsen number, Péclet number, Mach number and Brinkman number for the two last effects. However, the inclusion of pressure work in the analysis, associated with the viscous dissipation, leads to the appearance of the effect of shear work at the wall which is another important factor in the slip flow. The fundamental useful data concerning the heat transfer rates in microchannels may be obtained from the determination of the Nusselt number. Therefore, the need for accurate evaluation of this number is essential in the manufacture and design of microchannels. This generally requires a complete solution of the problem.

Although the effect of axial heat conduction may be often neglected in macroscale channels, for gas and liquids (with the exception of liquid metals); this is not justifiable in slip flow regime. This is because, for small scale geometries, the Péclet number will be small and the axial conduction plays a larger role. The effects of axial conduction for a fully-developed continuum flow, known as the extended Graetz problem, have been extensively investigated in the past (see for example [2–11]). This fundamental problem stills continue today to attract attention as an active area of research [12]. Recall that this problem reveals some difficulties such that the associated eigenvalue problem for the elliptic energy equation is non-selfadjoint. Therefore, it gives an incomplete set of eigenvalues that could, at least in principle, be complex. However, in series of innovative papers [4–6], Papoutsakis and Ramkrishna solved successfully this problem by decomposing the energy equation into a pair first order partial differential equations by using self-adjoint formalism. They showed that the analytical solution obtained is computationally simple and efficient as the solution for the parabolic problem. Others analytical studies related to the contribution of axial conduction in forced convection can be also found in Refs. [13,14]. Moreover, the combined effects of pressure work and viscous dissipation on Graetz problem for gas flows, in parallel-plate channels without axial conduction, have been investigated for the first time by Ou and Cheng [15]. It was found that, in continuum flow, these effects contribute significantly to the heat transfer.

The extended Graetz problem for slip flow with viscous dissipation has also widely been investigated in the literature (see for example [16–22]). These studies revealed that viscous dissipation plays also an important role at microscale due to the high velocity gradient. On the other hand, pressure work effect has been often neglected in the majority of previous studies considering the effect of viscous dissipation within the flow. However, for gas flows, contrary to liquids, it is shown that this factor is of the same order of magnitude as viscous dissipation and must therefore be taken also into consideration [15,23–25]. The role played by the shear work at the microscale was discussed by Lockerby and Reese [26] and considered also by Hadjiconstantinou [23] in the case of constant wall heat flux boundary conditions. A good literature review of experimental, numerical and analytical studies, concerning convective heat transfer in microchannels, with various geometries and different heat boundary conditions, can be found in the recent work of Colin [27]. Another important aspect encountered in gas microflows is the effect of compressibility. This effect has been also numerically examined [28]. This study shows that the compressible flow for large Mach number never reaches a fully developed state due to the effect of the acceleration of the gas. However, if the value of the Mach number along the channel is less than 0.2 and for relatively low Reynolds flows with small hydraulic diameter to length ratio, and when the pressure drop is less than 10% of the outlet pressure, the inertial effects can be then neglected [1,29,30]. Therefore, a fully developed state is possible provided that the above conditions are satisfied. However, as pointed out by Duan [30], for practical engineering applications, the fluid can be considered as incompressible for small pressure ratios, which is approximately consistent with experimental results of Jang and Wereley [31].

Despite numerous studies on the extended Graetz problem at microscale, to our knowledge, no investigation is known to have studied the heat transfer in microchannels having a heating section of finite size with the combined effect of viscous dissipation and pressure work under constant wall temperatures. The previous studies have often considered microchannel, having a very long heating section which is semi-infinite in length. However, for practical applications, this is not the most relevant configuration, since only a finite length of the heating section is allowed. This length becomes however another important factor in the analysis which may have also significant effect on the heat transfer characteristics.

The aim of the present contribution is to provide a purely analytical solution to the extended Graetz problem for a gas flow inside a micropipe as well as inside a parallel-plate microchannel in the slip flow regime with finite and semi-infinite heating sections and prescribed wall temperatures. The mathematical implication is that the microchannels are considered as infinite in extent and have two discontinuities in temperature at the