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# Effects of aspect ratio on natural convection of Bingham fluids in rectangular enclosures with differentially heated horizontal walls heated from below

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## ABSTRACT

In this analysis the effects of aspect ratio *AR* (ratio of enclosure height: length) on steady-state natural convection of yield stress fluids obeying the Bingham model within rectangular enclosures has been investigated for $1/4 \leqslant AR \leqslant 4$. A nominal Rayleigh number range $10^3 \leqslant Ra \leqslant 10^5$ (*Ra* defined based on the height) for a single representative value of nominal Prandtl number (i.e. $Pr = 500$) in a configuration with differentially heated horizontal walls subjected to constant wall temperatures with heated bottom wall has been considered. It has been found that the convective transport strengthens with increasing nominal Rayleigh number *Ra* for both Newtonian and Bingham fluids but the mean Nusselt number $\overline{Nu}$ for Bingham fluids remains smaller than the value obtained for Newtonian fluids for a given set of values of nominal *Ra* and *Pr* due to augmented viscous resistance arising from yield stress in Bingham fluids. For Bingham fluids $\overline{Nu}$ decreases with increasing Bingham number *Bn* (non-dimensional yield stress) and thermal transport becomes essentially conduction-driven for large values of *Bn*. The relative contribution of convection to the overall thermal transport diminishes (strengthens) with increasing (decreasing) *AR* for a given set of values of *Ra* and *Pr* for both Newtonian and Bingham fluids. Thus, the thermal transport is principally conduction dominated for tall enclosures. A detailed scaling analysis has been carried out to explain the effects of *AR*. This scaling analysis, in turn, has been utilised to propose a correlation, which has been demonstrated to predict $\overline{Nu}$ obtained from simulation data for $1/4 \leqslant AR \leqslant 4$, $10^3 \leqslant Ra \leqslant 10^5$ and $Pr = 500$.

## 1. Introduction

Natural convection in rectangular enclosures has been extensively analysed for Newtonian fluids and such studies have been comprehensively reviewed elsewhere [1,2]. In contrast only recently has a limited effort been directed to the study of natural convection of yield stress fluids in spite of its industrial importance in food, polymer and biological fluid storage and processing [3–15]. The Bingham fluid model is one of the simplest representations of yield stress fluids, which assumes linear strain rate dependence of shear stress under the yielded state but acts as a solid under an unyielded state [3]. Effects of yield stress on natural convection of Bingham fluids have been analysed by Vola et al. [4] in square enclosures with differentially heated vertical walls. This analysis was extended by Turan et al. [5] and Turan et al. [6] to investigate the effects of nominal Rayleigh, Prandtl and Bingham numbers (i.e. *Ra*, *Pr* and *Bn*) on the mean Nusselt number $\overline{Nu}$ for differentially heated vertical walls subjected to both constant wall temperature and wall heat flux boundary conditions respectively. It is well-known that the aspect ratio *AR* ($=H/L$ where *H* is the height and *L* is the enclosure length) plays a pivotal role in natural convection of Newtonian fluids in rectangular enclosures with differentially heated vertical side walls and interested readers are referred to references [16,17] for further information on this issue. Aspect ratio effects on natural convection of Bingham fluids in a differentially-heated side wall configuration has been numerically analysed for constant wall temperature [7] and constant wall heat flux boundary conditions [8]. Recently, Turan et al. [9] and Turan et al. [10] numerically analysed natural convection of Bingham fluids in square enclosures with differentially heated horizontal

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## Nomenclature

| | |
|---|---|
| $AR$ | aspect ratio [–] |
| $Bn$ | Bingham number [–] |
| $Bn_{max}$ | Bingham number at which the mean Nusselt number approaches unity [–] |
| $c_p$ | specific heat at constant pressure [J/kg K] |
| $f_2$ | function relating thermal and hydrodynamic boundary layers [–] |
| $g$ | gravitational acceleration [m/s$^2$] |
| $Gr$ | Grashof number [–] |
| $h$ | heat transfer coefficient [W/m$^2$ K] |
| $H$ | height of the enclosure [m] |
| $k$ | thermal conductivity coefficient [W/m K] |
| $L$ | length of the enclosure [m] |
| $m$ | stress growth exponent [s] |
| $m_{0\text{-}2}$ | correlation parameter [–] |
| $n_{0\text{-}2}$ | correlation parameter [–] |
| $Nu$ | Nusselt number [–] |
| $Nu_0$ | Nusselt number for Newtonian fluids [–] |
| $P$ | pressure [N/m$^2$] |
| $Pr$ | Prandtl number [–] |
| $q$ | heat flux [W/m$^2$] |
| $Ra$ | Rayleigh number [–] |
| $T$ | temperature [K] |
| $u_{i,}$ | $i$th component of velocity [m/s] |
| $V$ | dimensionless vertical velocity ($V = u_2\,L/\alpha$) [–] |
| $\vartheta$ | characteristic velocity [m/s] |
| $x_{i,j}$ | coordinate in $i$th and $j$th directions [m] |
| $x_{0\text{-}1}$ | correlation parameter [–] |
| $y_{0\text{-}1}$ | correlation parameter [–] |
| $\alpha$ | thermal diffusivity [m$^2$/s] |
| $\beta$ | coefficient of thermal expansion [1/K] |
| $\dot{\gamma}$ | strain rate [1/s] |
| $\delta,\delta_{th}$ | velocity and thermal boundary layer thickness [m] |
| $\theta$ | dimensionless temperature $\theta = (T - T_C)/(T_H - T_C)$ [–] |
| $\mu$ | dynamic viscosity [Ns/m$^2$] |
| $\mu_{yield}$ | yield viscosity [Ns/m$^2$] |
| $\nu$ | kinematic viscosity [m$^2$/s] |
| $\rho$ | density [kg/m$^3$] |
| $\tau_{ij}$ ($\tau$) | stress tensor (stress) [N/m$^2$] |
| $\tau_y$ | yield stress [N/m$^2$] |
| $\psi$ | stream function [m$^2$/s] |

*Subscripts*

| | |
|---|---|
| $C$ | cold wall |
| $H$ | hot wall |
| $CWT$ | constant wall temperature |
| $eff$ | effective |
| $w$ | wall value |

*Special characters*

| | |
|---|---|
| $\Delta T$ | the temperature difference hot and cold wall ($T_H - T_C$) [K] |
| $\overline{Nu}$ | mean Nusselt number [–] |

walls with heated bottom wall for constant wall temperature and wall heat flux boundary conditions respectively. Although superficially similar, natural convection in rectangular enclosures with differentially heated vertical walls is fundamentally different to that in the differentially heated horizontal wall configuration. The convection sets in once a finite temperature difference is established between the active walls in differentially heated vertical wall configuration, whereas a critical Rayleigh number needs to be surpassed for the onset of fluid motion for the differentially heated horizontal wall configuration. Thus the effects of $AR$ in rectangular enclosures with differentially heated horizontal walls are expected to be different from the observed aspect ratio effects in differentially heated sidewall configuration. The effects of partial spatial heating of the bottom wall and symmetrically cooled sidewalls for a viscoplastic Bingham fluid have recently been analysed numerically by Hassan et al. [11]. Recently, Sairamu et al. [18] concentrated on natural convection of Bingham fluids from a heated horizontal cylinder for different values of nominal Rayleigh, Prandtl and Bingham numbers, which demonstrated that the strength of convection diminishes with increasing Bingham number and the mean Nusselt number asymptotically approaches to the value obtained from pure conduction-driven transport for large values of Bingham number. Massmeyer et al. [19] recently numerically analysed thermal plumes for a well-known yield stress system ("Carbopol") using the Herschel-Buckley regularization, which showed that the flow resistance due to yield stress opposes the plume formation. The numerical findings of Massmeyer et al. [19] were found to be in good agreement with experimental results.

Although a number of recent analyses [12–15] studied the onset conditions for convection in Bingham fluids in rectangular enclosures, the present analysis deals with convection under conditions when flow is *already* present in the enclosure (i.e. the initial conditions for the Bingham simulations are not quiescent fluid but rather a state with convection already present). Until recently, experimental analysis of natural convection of yield stress fluids was largely lacking but two recent papers [14,15] have begun to address this deficit. These papers concentrated on the onset of fluid motion, i.e. the subcritical or supercritical nature of the instability itself, rather than in the heat transfer characteristics well beyond critical conditions which is the primary focus of the current paper. Natural convection of Bingham fluids in rectangular enclosures for different values of aspect ratio, heated from below, is yet to be analysed in detail and this gap in the existing literature is addressed here by carrying out two-dimensional steady-state numerical simulations within rectangular enclosures for aspect ratios ranging from 1/4 to 4 for a range of different values of nominal Rayleigh number $Ra = 10^3 - 10^5$ at a single representative Prandtl number $Pr = 500$ (for example $\sim$0.1% by mass Carbopol solution in water shows yield stress behaviour and has a Prandtl number of about 500 when the flow is approximated by the Bingham plastic model). The parametric analysis in the current study has been carried out for a range of Bingham number $0 \leqslant Bn \leqslant Bn_{\max}$ (where $Bn_{\max}$ is the Bingham number above which the mean Nusselt number attains a value of unity) because the heat transfer takes place purely due to thermal conduction for $Bn \geqslant Bn_{\max}$ and one obtains $\overline{Nu} = 1.0$. It was demonstrated in Ref. [9] that $Bn_{\max}$ depends on both $Ra$ and $Pr$ but the qualitative behaviour of heat and momentum transport is not affected by the value of nominal Prandtl number. Here $Bn_{\max}$ has been parameterised in terms of $Ra$, $Pr$ and $AR$ in such a manner that $Pr$ dependence of $Bn_{\max}$ reported earlier [9] can be accurately captured so it can be expected that the correlation proposed in this paper (see Eq. (24) later in this paper) is going to be valid for other values of Prandtl number within the range of Rayleigh number considered here (indeed it has been confirmed during the course of the present study that they work well for $Pr = 100$). The analysis by Bussse [20] indicated that steady two-dimensional (2D) solution does not exist for $Ra = 10^5$ in the case