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Analytical solution for combined heat and mass transfer in laminar falling film absorption with uniform film velocity - diabatic wall boundary

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ABSTRACT

In a previous study the Laplace transform has been introduced in order to solve the combined heat and mass transfer problem in absorbing laminar falling films with constant film velocity for an isothermal as well as an adiabatic wall boundary condition. It has been stated, that the Laplace transform basically allows to apply arbitrary wall boundary conditions in contrast to the Fourier method.

Therefore, in the present study a diabatic wall boundary condition is applied, which, as limiting cases, includes both the isothermal wall if the thermal resistance of the wall is zero as well as the adiabatic wall condition for an infinite thermal resistance of the wall.

Temperature and mass fraction profiles across the film as well as the evolution of the absorbed mass flux with increasing flow length are presented for two different thermal resistances of the wall and modified Stefan numbers. The results are compared to the limiting cases of the isothermal and the adiabatic wall boundary condition.

The present study offers an analytical solution with a more realistic boundary condition of a constant thermal resistance of the wall including the isothermal and the adiabatic wall boundary condition.

1. Introduction

Numerous analytical and numerical methods in order to solve the combined heat and mass transfer problem have been presented for various simplifications and assumptions. Thereby, the analytical solving procedures usually impose more or less extensive restrictions upon the physical model. For that reason and due to the wide availability of computing capacity for the last twenty-five years, numerical methods have prevailed in order to account for more realistic physical models, e.g. for the hydrodynamics of the film flow.

Although numerical solutions applied to the more complex physical problems provide detailed insights to the transfer mechanisms, they usually lack a convenient applicability to e.g. comprehensive absorption process simulation.

The reduced computational effort as compared to numerical methods, as well as the easy implementation of the analytical terms favour the analytical solutions to be used in absorption process design in order to identify crucial parameters. Nevertheless, the restrictions on the physical model due to the analytical solving procedure should be as minor as possible but still providing one comprehensive analytical solution covering the whole film flow range.

By means of the Laplace transform as a versatile analytical solving method, less restrictions on the thermal boundary conditions are imposed upon the present physical model and the problem is solved analytically for a diabatic wall boundary condition.

In order to evaluate the present solving method some of the established analytical and numerical methods are introduced briefly.

2. State of the art

As mentioned in the introduction, there is not only one combined heat and mass transfer problem, but multiple depending on the physical modelling, and the boundary conditions applied.

The present study is concerned with solving the following partial differential equations as a result from the differential energy and mass fraction balance for the film flow with uniform film velocity $\overline{u}$ [1]:

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Nomenclature

Dimensionless numbers

Le Lewis number $\left(\mathrm{Le} = a \cdot D^{-1}\right)$

$\tilde{\mathrm{Bi}}$ modified Biot number $\left(\tilde{\mathrm{Bi}} = U' \cdot \delta_{Film} \cdot \lambda_s^{-1}\right)$

$\tilde{\mathrm{St}}$ modified Stefan number $\left(\tilde{\mathrm{St}} = c_s \cdot \Delta T \cdot \Delta h_{abs}^{-1} \cdot \Delta c^{-1}\right)$

Greek letters

α heat transfer coefficient $\left[\mathrm{J\,s^{-1}\,m^{-2}\,K^{-1}}\right]$

Δ difference

δ thickness [m]

η dimensionless film thickness $\eta = y \cdot \delta^{-1}$

γ dimensionless absorbate mass fraction $\gamma = (c - c_0) \cdot (c_{eq} - c_0)^{-1}$

λ thermal conductivity $\left[\mathrm{W\,m^{-1}\,K^{-1}}\right]$

μ dimensionless mass flux

ρ density $\left[\mathrm{kg\,m^{-3}}\right]$

Θ dimensionless temperature $\Theta = (T - T_0) \cdot (T_{eq} - T_0)^{-1}$

ξ dimensionless flow coordinate $\xi = x \cdot \delta^{-2} u^{-1} a$

Latin letters

a thermal diffusivity $\left[\mathrm{m^2\,s^{-1}}\right]$

B upstream plate width [m]

c mass fraction (absorbate) $\left[\mathrm{kg\,kg^{-1}}\right]$

c specific heat capacity $\left[\mathrm{kJ\,kg^{-1}\,K^{-1}}\right]$

D mass diffusivity $\left[\mathrm{m^2\,s^{-1}}\right]$

h specific enthalpy $\left[\mathrm{kJ\,kg^{-1}}\right]$

i imaginary unit

k index

$\dot{M}$ mass flow $\left[\mathrm{kg\,s^{-1}}\right]$

$\dot{m}$ mass flux $\left[\mathrm{kg\,m^{-2}\,s^{-1}}\right]$

T temperature [K]

u mean streamwise film velocity $\left[\mathrm{m\,s^{-1}}\right]$

U' partial heat transmission coefficient $\left[\mathrm{J\,s^{-1}\,m^{-2}\,K^{-1}}\right]$

x streamwise direction [m]

y transverse direction to film flow [m]

z complex Laplace variable

Sub-/Superscripts/Symbols

0 inlet values

abs absorption

eq equilibrium (at inlet condition)

ext external fluid

film film

i interface

s solution

st short term

W wall

$$\overline{u}\frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial y^2}, \tag{1}$$

$$\overline{u}\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial y^2}. \tag{2}$$

Grigor'eva and Nakoryakov [2,3] introduced the Fourier method in order to solve (1) and (2) analytically. Although the reported mathematical instabilities of the solutions obtained with the Fourier method [3] can be overcome by rearranging the implicit tangent function [4] extending this solution to the whole film flow range, the boundary condition for the temperature profile at the wall is restricted to either isothermal or adiabatic. This restriction originates in the orthogonality relation, which is indispensable in order to apply the Fourier method.

Grossman [5] extended the model of Grigor'eva and Nakoryakov by applying a steady state parabolic film velocity profile, $u(y)$, presented by Nusselt [6] instead of $\overline{u}$ in (1) and (2). By means of the Fourier method, Grossman obtained ordinary differential equations with non-constant coefficients and he solved them using recursion formulas resulting in infinite power series as eigenfunctions. Due to problems of convergence with these power series, Grossman stated that his solution only converges for moderate and large values of the dimensionless flow length. Moreover, Grossman's analytical solution is restricted to wall temperatures that equal the film inlet temperature. Due to the drawbacks of his analytical solving method, Grossman employed numerical techniques in order to solve the combined transfer problem using a short term analytical solution at the inlet for stability reasons.

Wekken and Wassenaar [7] applied the finite element method to solve the system of partial differential equations numerically for a diabatic wall boundary condition as well as for an unidirectional diffusion model as interface boundary condition. Nevertheless, they assume equimolar counter diffusion within the film neglecting the induced convective flow perpendicular to the film flow due to unidirectional diffusion.

Brauner [8] set up a less restricted physical model taking into account this convective flow perpendicular to the film flow, but again only for either the isothermal or the adiabatic wall boundary condition. She also accounted for a variable film thickness due to absorption and hence, a varying film velocity in her general physical film model. In order to solve the system of partial differential equations by an integral approach, she divided the film into different zones. e.g. the low penetration zone, which for infinite dilution is identical with the short term approach for the unaffected interface boundary of Nakoryakov et al. [9]. Brauner obtained analytical expressions solving the transfer problem only for some limiting cases e.g. the low penetration zone or the isothermal absorption. For the other downstream zones and cases she used numerical integration techniques.

Conlisk [10] further simplified the combined problem for the isothermal wall boundary condition by assuming a linear temperature profile "very soon after entering the tube" and consider the mass transfer resistance to be decisive. He obtained analytical expressions for the absorbed mass flux assuming that the mass transfer only takes place in a thin interface boundary layer, which restricts this solution to moderate flow lengths only.

In 2001, Killion and Garimella [11] presented a thorough review of the multiple models and solving methods for the combined heat and mass transfer problem.

In the present study, there are neither restrictions in the range of validity nor the choice of the wall boundary condition for the temperature profile is restricted, since the Laplace transform is applied in order to solve the partial differential equations analytically. Hence, a wall boundary condition of the third kind, the diabatic wall, is applied. By means of a method to find the inverse Laplace transform as presented by Baehr [12], the solution in the real domain is obtained.

The film model and its simplifying assumptions have been derived in [1] and therefore the present study directly starts with the dimensionless inlet and boundary conditions and the application of the Laplace transform to the dimensionless partial differential equations for energy (3) and absorbate mass fraction (4):