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Natural convection in square enclosure induced by inner circular cylinder with time-periodic pulsating temperature

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ABSTRACT

The periodic unsteady natural convection flow and heat transfer in a square enclosure containing a concentric circular cylinder is numerically studied. The temperature of the inner circular cylinder fluctuates periodically with time at higher averaged value while the temperature of the enclosure keeps lower constant, and the natural convection is driven by the temperature difference. The two-dimensional natural convection is simulated with high accuracy temporal spectral method and local radial basis functions method. The Rayleigh number is studied in the range $10^3 \leqslant \mathrm{Ra} \leqslant 10^6$, the temperature pulsating period ranges from 0.01 to 100 and the temperature pulsating amplitudes are $a = 0.5$, 1.0 and 1.5. Numerical results reveal that the fluid flow and heat transfer is strongly dependent on the pulsating temperature of inner cylinder. Comparing with the steady state natural convection, the heat transfer is enhanced generally for the time-periodic unsteady natural convection, and the local maximum heat transfer rate is observed for $\mathrm{Ra} = 10^5$ and 10^6. Moreover, the phenomenon of backward heat transfer is discussed quantitatively. Also, the influence of pulsating temperature on the unsteady fluid flow and heat transfer are discussed and analyzed.

1. Introduction

Fluid flow and heat transfer of the natural convection in an enclosure has been a subject of considerable interest to the scientists and researchers for several decades due to its numerous engineering applications such as compact heat exchangers, nuclear and chemical reactors, solar collection systems and electronic equipment cooling. The natural convection induced by a heated circular cylinder inside square enclosure might be complex and more realistic. Therefore, a large number of investigations on this issue have received much attention of the researchers and engineers.

Moukalled and Acharya [1] numerically investigated the natural convection in a square enclosure with a heated horizontal circular cylinder concentrically placed inside. They considered the effects of aspects ratios between square enclosure and circular cylinder and the Rayleigh number. Their numerical results show that the convection contributing to the total heat transfer decreases with the increase of aspect ratio. With the differential quadrature method, Shu et al. [2] numerically studied the natural convection for the analysis of flow and thermal field at different concentricities and angular positions of the transformed coordinate. They found that the global circulation, flow separation and the top space between the square outer cylinder and the circular inner cylinder have significant effects on the plume inclination.

The natural convection in a concentric annulus was simulated by Shu and Zhu [3]. They found that both aspect ratio and Rayleigh number are critical to the patterns of flow and thermal fields. Kim et al. [4] and Hussain and Hussain [5] numerically investigated the influence of vertical locations of inner cylinder on the steady natural convection. They reported that the number, size and formation of the vortices and heat transfer rate are strongly dependent on the Rayleigh number and the position of the inner circular cylinder. Lee et al. [6] studied the effect of horizontal and diagonal locations of the inner circular cylinder on the natural convection. The effect of enclosure inclination on the natural convection has been numerically studied by Park et al. [7]. Above studies are about the isotherm boundary conditions on the cylinder and enclosure, the natural convection with various thermal boundary conditions are also investigated [8–9].

The studies discussed above are focused on steady natural convection, and it might bifurcate to unsteady as the Rayleigh number increases. To explore this bifurcation, Angeli et al. [10] simulated

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Nomenclature

a	temperature pulsating amplitude
f	thermal conductivity of fluid
g	gravitational acceleration
h	local heat transfer coefficient
L	length of square enclosure
n	index of iteration
NT	number of equispaced time intervals
Nu	overall Nusselt number
Nu_l	local Nusselt number, $hd/f = -\partial\theta/\partial\mathbf{n}$
$\mathbf{n}$	unit normal vector
P	temperature pulsating period
Pr	Prandtl number, ν/α
r	radius of circular cylinder
Ra	Rayleigh number, $g\beta\Delta\theta L^3/\nu\alpha$
t	dimensionless time
T	temperature
u, v	dimensionless velocity components
$\mathbf{u}$	velocity vector, (u, v)
x, y	directions of Cartesian coordinate

Greek symbols

α	thermal diffusivity of fluid
β	coefficient of volume expansion
ν	kinematic viscosity of the fluid
θ	dimensionless temperature
τ	pseudo time
ψ	stream function
ω	vorticity

Subscripts

c	cold
h	hot
l	local

Superscript

–	period averaged

the natural convection and observed three flow regimes including stable symmetric, non-symmetric steady-state and unsteady regimes as the Rayleigh number increases. Yoon et al. [11] performed the simulations at the Rayleigh number of 10^7 for different vertical locations. They found that the natural convection bifurcates from steady to unsteady state according to the location of the inner circular cylinder. Recently, Kang et al. [12] numerically studied the natural convection as the inner heated circular locates along a horizontal or diagonal line in square enclosure at Ra = 10^7. They confirmed that the natural convective heat transfer and fluid flow bifurcate from steady to unsteady or from unsteady to steady at critical positions with both the horizontal and diagonal lines.

The unsteady natural convection heat transfer driven by other excitations might get some new phenomena. The usually employed excitation is sinusoidally varying sidewall temperature, which has been investigated extensively in unsteady natural convection in a cavity since first studied by Kazmierczak and Chinoda [13]. They simulated the natural convection in a square cavity, in which the hot sidewall temperature varies periodically and the cold opposite wall keeps constant. They studied the effects of the hot wall temperature pulsating amplitude ($0.2 \leqslant a \leqslant 0.8$) and period ($0.005 \leqslant P \leqslant 0.02$) and found that the transient buoyancy-driven flow is periodic in time and back heat transfer through the warm sidewall is observed. Kalabin et al. [14] numerically studied the periodic natural convection in a tilted square cavity, in which the temperature of cold wall oscillates with $a = 5$ and the hot wall keeps constant. They demonstrated that there is possibility that heat is transferred from the cold to the hot wall due to time-periodic flow when the oscillating amplitude is greater than the difference between hot wall temperature and time-averaged cold wall temperature. There are many studies on the periodic unsteady natural convection in an enclosure driven by time-periodic pulsating temperature [15,21–24].

Inspired by the natural convection heat transfer driven by is sinusoidally varying sidewall temperature, recently, Roslan et al. [16] investigated the unsteady natural convection induced by a sinusoidally heated circular cylinder in a square enclosure numerically with COMSOL. Within their considered parameters $0.1 \leqslant a \leqslant 0.9$ and $0.04 \leqslant P \leqslant 0.4$, the fluid flow and heat transfer is definitely time periodic and the heat transfer rate follows a sinusoidal law; moreover, the enhancement of heat transfer is observed. They also mentioned the negative instantaneous heat transfer rate at large temperature pulsating amplitude. However, confined to their parameters, the interesting phenomenon of backward heat transfer corresponding to negative heat transfer rate has not been discussed. To explore this phenomenon and related fluid flow and heat transfer process, this paper presents a comprehensive study on the time periodic natural convection induced by the sinusoidal temperature of inner circular cylinder. The high accuracy temporal pseudospectral method which is very suitable for time periodic problem and local radial basis functions method are developed for the numerical simulations. The parameters are chosen as following, the Rayleigh number $10^3 \leqslant \text{Ra} \leqslant 10^6$, the temperature pulsating period and amplitude are $0.01 \leqslant P \leqslant 100$ and $a = 0.5$, 1.0 and 1.5 respectively, while the Prandtl number is fixed at Pr = 0.71 and the ratio between the radius of circular cylinder and length of the square enclosure is kept as 0.2.

2. Mathematical model and numerical methods

2.1. Governing equations and boundary conditions

The non-dimensional governing equations for the two-dimensional incompressible natural convective flow based on Boussinesq approximation can be expressed in the vorticity–stream function formulation as

$$\begin{aligned} &\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=\omega \\ &\frac{\partial\omega}{\partial t}+u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}=\Pr\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)-\Pr\,\text{Ra}\frac{\partial\theta}{\partial x} \\ &\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2} \end{aligned} \quad (1)$$

where ψ represents stream function, ω denotes vorticity and θ is the temperature. The reference variables used in the nondimensionalization are: the length of square enclosure L, the temperature difference between inner circular cylinder and outer square enclosure $\Delta T = \overline{T}_h - T_c$. Prandtl number is defined as $\Pr = \nu/\alpha$, and Rayleigh number is defined as $\text{Ra} = g\beta L^3\,\Delta T/\nu\alpha$. Here ν, α, β and g are the fluid thermal diffusivity, kinematic viscosity, volume expansion coefficient and gravitational acceleration respectively. The velocity components u and v can be obtained by the stream function ψ.