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Thermal behavior of solid particles at ignition: Theoretical limit between thermally thick and thin solids

A. Lamorlette [a,*], F. Candelier [b]

[a] Aix-Marseille Université, CNRS, Centrale Marseille, M2P2 UMR 7340, 13 451 Marseille, France
[b] Aix-Marseille Université, CNRS, IUSTI UMR 7343, 13 451 Marseille, France

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ABSTRACT

This paper deals with thermal behaviors of solid particles at ignition in attempting to theoretically delineate transition between thermally thick and thin behavior when a solid target is exposed to a radiant heat flux. In order to evaluate classical asymptotic relation accuracy and limiting range, models are developed for finite-depth target in both Cartesian and cylindrical coordinates, allowing to enhance asymptotic relations. Comparison between finite-depth target solutions and asymptotic solutions finally provides a mapping which allows the suited relation for ignition time calculation to be determined, regarding ignition conditions. This mapping then suggests some interesting consequences on forest fuel ignition and fire propagation modeling, since asymptotic models seem to overlap on large regions.

1. Introduction

In the frame of fire safety, a key point is the modeling of flaming ignition time of solid material. A first approximation, often called the classical ignition theory, consists in considering ignition as two separated mechanisms: Heat and pyrolysis of the solid followed by chemical reactions in the gas phase, due to a surface absorption of the incoming radiation. If the pyrolysis gas flow is weak and the oxygen concentration is normal, ignition is expected to occur when the solid surface temperature reaches pyrolysis temperature. Indeed, it is usually assumed that chemical time and mixing time are much smaller than pyrolysis time for piloted flaming ignition. Ignition time then corresponds to the time needed by the solid to heat until it begin pyrolysing. This leads to model ignition as a temperature raise process in the solid material as suggested by [1].

Despite the fact that considering an inert solid until pyrolysis temperature is reached is a strong and questionable assumption, especially when the solid has reached pyrolysis temperature but has not yet ignite (see [2]) or when the solid undergo dehydration, this approach provides us with a first insight on the time needed to reach flaming ignition which presents a satisfactory experimental validation. Rigorously, note that according to [3–5], a mixing time should be added to the heating time in order to provide the ignition time for the smaller target (i.e. thermally thin target) [3] or for the higher heat fluxes [5]. However, experimental results of [3,6] suggest that this mixing time remains negligible in comparison with the heating time, whereas in [4], experimental results show that the mixing time cannot be neglected for some class of materials. Moreover, when the heat flux is too high, radiation must be modeled considering in-depth absorption as shown in [7], what implicitly suggests a thermally thick behavior. Hence, this study focuses on the heating time, what sounds acceptable to discriminate the thermal behavior at ignition for many class of materials.

For thermally thick surfaces, ignition is thus generally modeled as a one-dimensional heat conduction problem as proposed by [2]. For thermally thin particles (i.e. when the temperature gradient inside the particle can be neglected), a zero-dimension heat transfer problem is considered as suggested in [6]. To our knowledge, the limit at ignition between thermally thick and thermally thin behaviors has not yet been clearly established from a theoretical point of view. Indeed, the following statement can be found in a major reference on this topic, the SFPE Handbook of fire protection engineering [2]: *"A simple criterion based on the Biot number (Bi) is generally used for the purpose of establishing if a material is thermally thin or thick. The Biot number is defined as $Bi = hL/\lambda$ where h is a global heat transfer coefficient (W/m^2K) and λ is the thermal conductivity (W/mK). If Bi is much less than 1 then temperature gradients inside the solid are negligible; whereas if the Biot number is not much smaller than unity, then temperature gradients need to be considered."* The physical meaning of the suggested criterion is nevertheless questionable. Indeed, in most flaming ignition situations, the heat flux is radiative, therefore the criterion should be based on a

* Corresponding author. Tel.: +33 491 113 811; fax: +33 491 118 502.
E-mail address: aymeric.lamorlette@univ-amu.fr (A. Lamorlette).

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radiative Biot number as suggested in [8] for studying woody particle pyrolysis. A first attempt to delineate transitions between the two asymptotic thermal regimes has been realized by [9] where asymptotic solutions for thermally thick and thin samples are compared to experimental data, providing insights on the thermal behavior at ignition. However, no theoretical limit can be established using this approach. For instance, [9] claims that 6 mm thick wood sample ignition time can be accurately estimated using either thermally thick or thin model without providing any theoretical explanation. A deeper investigation is performed by [10] where analytical corrections are suggested for the asymptotic cases, developed on the basis of heat diffusion solutions suggested in [11]. These solutions allow to emphasize a similar behavior for the two asymptotic cases on a given range of intermediate thermal conditions, explaining the behavior of the 6 mm thick wood sample previously mentioned. Therefore, it seems relevant to us to investigate this peculiar behavior for any particle size and especially for smaller particles such as pine needles, small branches and twigs composing forest fuel layers. Pine needles for instance seem to exhibit the same hybrid behavior as the 6 mm thick wood sample. Indeed, attempt to develop ignition time model for forest fuel layers are often assuming that particles are thermally thin, as in [12] for instance. However, experimental results of [13] suggest that the layer or the particles composing the layer are thermally thick. A surprising result is that, though they are based on opposite assumptions, these two models provide us with accurate ignition time estimation for many experimental data such as those of [12,13] on a broad range of particle size and incoming heat flux. Hence, these experiments do not allow to delineate the thermal behavior of the particles composing the fuel layer. Therefore, this study aims at clearly defining, according to theoretical means, the thermal limit between thick and thin behaviors using the classical ignition theory.

This analysis is first performed in the simplest case, that is a finite-depth slab with one side exposed to a radiant heat flux and both sides loosing heat by convection and radiant re-emission through a total heat transfer coefficient h, considering a linear approximation for radiant re-emission, what can be considered as an acceptable assumption, according to [5,13]. Exact solution for this heat diffusion problem can be obtained (see [11]), allowing to easily estimate asymptotic solution accuracy. Comparisons between asymptotic and exact solutions for a broad range of dimensionless heat flux and Biot number then allow to map the thermal behavior of such target during ignition, providing a criterion to define which asymptotic relation is suited to estimate ignition time. The same analysis is then performed for an infinite cylinder under a collimated radiant heat flux heating inhomogeneously half the cylinder surface, in order to take into account curvature effects both for the incoming heat flux and for the heat diffusion. This peculiar heat diffusion problem sounds relevant for ignition studies, however it is too specific to be described in classical heat transfer theory books. Therefore, the solution is fully presented in this study before comparing it to those derived asymptotic solutions.

This paper is then organized as follows: A first section is dedicated to the one-dimensional finite-depth heat diffusion problem where an analytical solution is provided. A second section consists in comparing the previous solution with the classical thermally thick and thin solutions existing in the literature and also in suggesting new, more accurate asymptotic solutions in both cases of thermally thick and thin material. This section also summarizes the results presented in this study for a finite-depth slab in suggesting a mapping of the thermal behavior at ignition regarding both convective (or classical) and radiative Biot number (the latter can also be defined as a dimensionless heat flux). The same analysis is then conducted on an infinite cylinder under a collimated radiant heat flux. Finally, a discussion suggests how ignition of particles should be modeled regarding Biot numbers, allowing to draw consequences on modeling of forest fuel layer ignition and fire propagation.

2. Mathematical formulation: finite-depth conduction problem

A one-dimensional inert solid material of depth L is considered with one side exposed to an incoming heat flux $\phi(t)$. The heat conduction equation is now expressed for the solid target, introducing $\theta(x,t) = (T(x,t) - T_0)/(T_{ig} - T_0)$ where $T(x,t)$ is the solid temperature, T_0 is the initial temperature of both the solid and its surrounding air and T_{ig} is the solid ignition temperature; λ is the solid heat conductivity, ρ its density, Cp its specific heat and h is a total heat transfer coefficient. The following quantities are thus suggested to normalize equations:

$$Bi = \frac{hL}{\lambda}, \tag{1}$$

$$\Phi = \frac{\phi L}{\lambda(T_{ig} - T_0)}, \tag{2}$$

$$t^* = \frac{t\lambda}{\rho C p L^2}; \quad x^* = \frac{x}{L}. \tag{3}$$

The dimensionless heat diffusion problem thus reads (superscripts $*$ are removed for clarity):

$$\frac{\partial\theta(x,t)}{\partial t} = \frac{\partial^2\theta(x,t)}{\partial x^2}, \tag{4}$$

with the following boundary conditions:

$$-\frac{\partial\theta(x,t)}{\partial x} = \Phi - Bi\theta(x,t), \quad x = 0 \tag{5}$$

$$-\frac{\partial\theta(x,t)}{\partial x} = Bi\theta(x,t), \quad x = 1. \tag{6}$$

$$\theta(x,0) = 0, \quad \forall x \tag{7}$$

In order to get a general solution of this equation for an arbitrary function $\phi(t)$, let us first consider the response of Eq. (4) to a sudden constant heat flux $\phi(t) = \phi H(t)$, where $H(t)$ is the Heaviside step function. The classical following methodology is now applied to solve these equations, according to [11]: A steady-state solution θ_0 is first sought for Eqs. (4)–(6), then a time-dependent perturbation θ_1 is sought using separation of variables, with $\theta = \Phi\cdot(\theta_0 + \theta_1)$ using Eq. (4) linearity. The problem admit the following solution $\theta_1 = e^{-X^2 t}(C_1\cos Xx + C_2\sin Xx)$. Introducing θ_1 in Eqs. (5) and (6) provides the trivial solution $C_1 = C_2 = 0$ since the problem relative to the temperature perturbation is homogeneous. Hence, the only way to obtain non-zero solution is to set the system determinant to zero. Thus Eqs. (5) and (6) lead to the following characteristic equation, according to [11]:

$$\tan X_n = \frac{2BiX_n}{X_n^2 - Bi^2}. \tag{8}$$

This equation is not solved in [11]; Indeed, zero determination for this equation requires numerical means. However in this article, solutions will be suggested for the first zero as described later, allowing to define a fully analytical expression for large time scale. θ_1 then reads as:

$$\theta_1 = \sum_{n=1}^{\infty} C_n e^{-X_n^2 t}\left(\cos X_n x + \frac{Bi}{X_n}\sin X_n x\right), \tag{9}$$

with $\theta_1(t=0) = -\theta_0$, leading to:

$$\sum_{n=1}^{\infty} C_n e^{-X_n^2 t}\left(\cos X_n x + \frac{Bi}{X_n}\sin X_n x\right) = -\frac{Bi(1-x)+1}{Bi(Bi+2)}. \tag{10}$$