International Journal of Heat and Mass Transfer 82 (2015) 348–356

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Evolution of accelerated laminar boundary layer subjected to isothermal wall heating

Alexey Yu. Sakhnov

Kutateladze Institute of Thermophysics SB RAS, Lavrentyev ave, 1, Novosibirsk, Russia

ARTICLE INFO

Article history:
Received 12 August 2014
Received in revised form 18 November 2014
Accepted 19 November 2014
Available online 6 December 2014

Keywords:
Boundary layer
Favorable pressure gradient
High temperature difference
Isothermal condition
Velocity overshoot

ABSTRACT

The paper reports on numerical simulation of a laminar near-wall flow subjected simultaneously to a favorable pressure gradient and wall heating at a constant temperature. An important feature of such a flow is an overshoot of the streamwise velocity profile when the velocity within the boundary layer exceeds the velocity in the external main flow. It is shown that the overshoot appears due to the simultaneous action of wall heating and acceleration. The investigation shows that the maximum velocity at a finite Reynolds number arises when the acceleration parameter is positive and the ratio of the wall temperature and the free stream temperature is greater than unity. The overshoot causes an increase of the skin-friction coefficient and the Stanton number. In addition, it leads to a negative momentum thickness. The article demonstrates the applicability of the approximate formula by Volchkov (2006) for the evaluation of the maximum velocity. Based on the own recent research and on an overview of the published literature on similar and related flows, it is concluded that the main prerequisite for a velocity overshoot is an input of energy from a bounding wall.

1. Introduction

An overshoot in the streamwise velocity profile is a phenomenon of near-wall flows, when the maximal value of the velocity is located within the boundary layer. It can occur at different conditions: in forced and mixed convection, heterogeneous injection, combustion, moving wall, pulsating flow, wall jet and others. The literature on the velocity overshoot is abundant and here only a brief overview will be given of the research relevant to the present investigations.

Convection along vertical heated walls is a common example of flows exhibiting velocity overshoot. Seetharam et al. [1] examined forced and mixed convection on a vertical flat plate heated at a temperature 20 K higher than the ambient or aiding flow temperature. The observed maximum velocity was 17% higher than that of the aiding flow, which was 0.15 m/s.

Abedin et al. [2] performed direct numerical simulation (DNS) of mixed convection of water along a heated vertical flat plate with aiding and opposing free streams. The authors focused primarily on the transition to turbulence. However, their velocity profile showed that the aiding free stream does not affect on overshoot, but the opposing flow reduces the maximum value of the velocity.

Singh et al. [3] investigated mixed convection over a vertical wedge with permeable surface and showed that suction leads to a reduction of the overshoot and injection augments it. In addition, they considered the effect of the Prandtl number in the range of 0.7–7.0 and found that the overshoot existed only for Pr = 0.7.

Subhashini et al. [4] studied simultaneous effects of thermal and concentration diffusions on a mixed convection boundary layer flow over a permeable surface and obtained similar results on the Prandtl number effect. However, they detected a significant overshoot in buoyancy assisted flow only at low Prandtl numbers. Their paper demonstrates that the overshoot grows with the increased difference in the concentrations of the sucked or injected fluid in the main flow and near the permeable wall.

Patil et al. [5] described the effects of suction and blowing through the permeable wall on the overshoot in a mixed convection boundary layer. In agreement with the results published in [3], these authors also found that suction decreases the overshoot, and injection increases it up to 35%. The authors said that the overshoot is detected just for the buoyancy assisting flow.

The overshoot also appears in forced convection flow with heterogeneous injection. Fox and Libby [6] theoretically analyzed air flow in the vicinity of the stagnation point with helium blowing at different wall temperature. They found that in the case of a foreign-gas blowing, the condition $\rho_e > \rho_w$, proposed earlier for homogeneous flows by Baxter and Flügge-Lotz [7], is not sufficient for an overshoot. In addition, the velocity overshoot could reach

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.11.068
0017-9310/© 2014 Elsevier Ltd. All rights reserved.

Nomenclature

c_f skin-friction coefficient
c_p gas specific heat at constant pressure [J/(kg deg)]
$K = \left(\mu/\rho U_e^2\right) dU_e/dx$ stream acceleration parameter
L length of wall [m]
Pr Prandtl number
Re_x Reynolds number, based on streamwise coordinate x
Re_y wall-distance Reynolds number, based y-coordinate
Re^{**} momentum thickness Reynolds number
$\mathrm{Re}_{\mathrm{int}}$ integral Reynolds number
Re_{x0} Reynolds number, based on fixed initial velocity U_0
$\mathrm{St} = q_w/\rho_e U_e c_p \Delta T$ Stanton number
T temperature [K]
U, V velocity components in the x, y directions respectively [m/s]
x, y streamwise and normal coordinates relative to surface of streamlined body [m]

Greek symbols
α the convergence angle of top plane of channel [°]
δ thickness of hydrodynamic boundary layer [m], $U/U_e = 0.995$ or 1.005
$\delta^* = \int_0^\infty (1 - \rho U/\rho_e U_e) dy$ displacement thickness [m]
$\delta^{**} = \int_0^\infty \rho U/\rho_e U_e (1 - U/U_e) dy$ momentum thickness [m]
δ_T thickness of thermal boundary layer [m], $\Theta = 0.995$
$\delta_T^{**} = \int_0^\infty \rho U/\rho_e U_e (1 - \Theta) dy$ enthalpy thickness [m]
λ heat conductivity [W/(m K)]
μ dynamic viscosity [Pa s]
$\Theta = (T - T_w)/(T_e - T_w)$ dimensionless temperature
ρ density [kg/m^3]
$\psi = T_w/T_e$ temperature ratio

Subscripts
0 flow quantities at the start of flow
e flow quantities in external flow
max flow quantities at the point of maximum velocity
w parameter at the wall

100% for certain values of the helium blowing rate through the surface.

Gershbein [8] studied numerically and analytically an accelerated flow of a gas mixture around a body. The gas mixture consisted of hydrogen, nitrogen, and carbon dioxide. A similar gas mixture was injected into the flow through the body surface, but with other concentrations of the mixture components. Two types of flow with different density ratios were considered. The first flow was with $\rho_e/\rho_w = 0.28$; here, despite the presence of a favorable pressure gradient, the velocity profiles were less filled than the Blasius profile [9]. In the second flow considered, the density of blown mixture was 2.25 times lower than the gas density in the free stream. The streamwise velocity in the boundary layer was 45% greater than the main-stream velocity at the highest value of the favorable pressure gradient.

Several researchers revealed overshoots in flows with combustion and favorable pressure gradient. Ueda et al. [10] found that the velocity in the accelerated boundary layer with methane combustion exceeds the external flow velocity by 30%. They proved a negligible influence of injection on the velocity distribution in their experiments. These authors noted that the free stream acceleration is one of the main factors affecting the overshoot.

Ramachandra and Raghunandan [11] reported experimental data on n-pentane combustion in a confined channel flow. The excess over the main-stream velocity was up to 35%. The investigators argue that the overshoot increases with the increase of the injection rate and the distance from the leading edge of the porous plate.

Hirano et al. [12] studied accelerated boundary layer with combustion of methane and propane. The overshoot could reach 50% for a flow with propane combustion. It was concluded that maxima of the temperature and streamwise velocity do not always coincide with each other.

Boyarshinov et al. [13] examined combustion of ethanol in the boundary layer with a favorable pressure gradient. For the acceleration parameter $K = 19 \times 10^{-6}$ the velocity overshoot was 85% at the distance from the leading edge of $x = 0.32$ m. The authors showed that the combined influence of chemical reaction and flow acceleration leads to an increase of heat transfer.

The maximum of the flow velocity can exist also in pulsating flows and near the moving walls. One example is reported by O'Brien and Logan [14] in a pulsating boundary layer between two parallel plates. Here, the overshoot was small, about 7%.

This article presents some results of a numerical study of laminar boundary layers on a wall heated at a constant temperature, subjected to a favorable pressure gradient. The aim is to show the influence of the simultaneous mainstream acceleration and wall heating on the appearance and development of the velocity overshoot. In addition, the paper discusses the combined effect of these factors on the skin-friction, heat transfer, velocity and temperature distributions, as well as on some integral parameters of the boundary layer. The paper consists of three main sections: modeling and computational procedure, results, and conclusion. The first section describes the flow configuration considered, equations, boundary conditions and the solution method. The second section presents and discusses the results for the velocity, temperature and density fields and the features of the integral parameters. The conclusions summarize the main findings of the study.

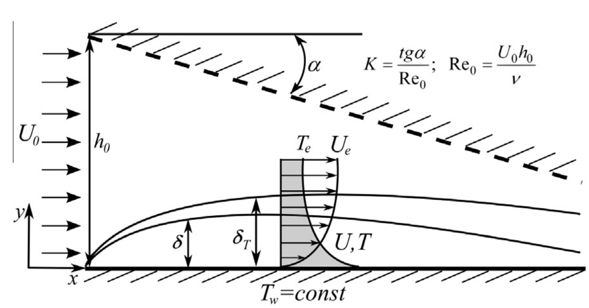

Fig. 1. Schematics of the flow considered.