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Volume-averaged macroscopic equation for fluid flow in moving porous media

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ABSTRACT

Darcy's law and the Brinkman equation are two main models used for creeping fluid flows inside moving permeable particles. For these two models, the time derivative and the nonlinear convective terms of fluid velocity are neglected in the momentum equation. In this paper, a new momentum equation including these two terms are rigorously derived from the pore-scale microscopic equations by the volume-averaging method. It is shown that Darcy's law and the Brinkman equation can be reduced from the derived equation under creeping flow conditions. Using the lattice Boltzmann equation (LBE) method, the macroscopic equations are solved for the problem of a porous circular cylinder moving along the centerline of a channel. Galilean invariance of the equations are investigated both with the intrinsic phase averaged velocity and the phase averaged velocity. The results demonstrate that the commonly used phase averaged velocity cannot be considered, while the intrinsic phase averaged velocity should be chosen for porous particulate systems. In addition, the Poiseuille flow in a porous channel is simulated using the LBE method with the improved equations, and good agreements are obtained when compared with the finite-difference solutions.

1. Introduction

The motion of permeable particles in a fluid has long received considerable attention in many fields such as colloid science, chemical, biomedical and environmental engineering. Because the fluid can penetrate into a permeable particle, there is a flow relative to the rigid skeleton of the porous medium. The hydrodynamic fields outside and inside the particles need to be treated together, which differs much from those of solid impermeable particles [1]. Numerous studies have been devoted to the understanding of transport phenomena in moving porous media for applications, such as sedimentation, agglomeration, flotation and filtration.

In order to obtain the fluid velocity within a permeable particle, conservation equations which accurately govern the fluid flows are required for the permeable region [2]. Under creeping flow conditions and considering the resistance force from the solid surface of moving porous media, two models for the fluid motion within porous media are commonly employed in the literature, i.e., Darcy's law and the Brinkman equation. Using the Stokes equation and Darcy's law, Payatakes and Dassios [3] investigated the motion of a porous sphere toward a solid planar wall. Later, Burganos et al. [4] provided a revision to their work with respect to the drag force exerted on the permeable particle. Owing to the negligence of the viscous dissipation term, the momentum equation for the interior fluid of a porous medium involves only first-order spatial derivatives in Darcy's law, while the momentum equation for the outside fluid includes spatial derivatives up to the second-order. This brings out a general fact that Darcy's law is confined to the case that the permeability of the porous medium is sufficiently low. Meanwhile, the continuity in both the fluid velocity and the stress at the interface between the permeable medium and the exterior fluid are not guaranteed [1]. Complementary boundary treatment is hence needed to satisfy the continuity at the interface of a moving porous medium [5–8]. In contrast, in the Brinkman equation, the velocity-gradient term corresponding to viscous dissipation of the fluid within the porous medium is incorporated in the momentum equation, and the continuity of the fluid velocity is fulfilled at the surface of the porous body. From this point of view, the Brinkman equation is more suitable than Darcy's law in the porous particulate systems.

Based on the Brinkman equation, numerous theoretical and computational studies have been conducted concerning moving

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permeable particles. For example, Jones [9–11] calculated the forces and torques on moving porous particles with a method of reflection, and some studies were also carried out to investigate the suspension flow of porous particles using the Brinkman equation with other accompanying methods [1,12–15]. Recently, the Brinkman model was also employed to study the hydrodynamic motion and interactions of composite particles [5,16–18].

In all of the aforementioned studies, the flow inside a moving permeable particle is described using either Darcy's law or the Brinkman model. In these two models, the transient term and the nonlinear inertial term are not included in the momentum equation. It follows that there is no mechanism to treat the unsteady evolution of flow fields, and the flow Reynolds number for fluid flows must be kept sufficiently small. However, as noted by Wood [19], the inertial effect on the flow and transport in porous media should be considered in many practical applications. To the best of our knowledge, no theoretical and numerical works have been developed to address this limitation. A new model is therefore desired for moving porous particulate systems at finite flow Reynolds numbers.

On the other hand, due to the complexity of internal geometries and interfacial structures, it is impractical to solve the microscopic conservation equations inside the pores. A preferable approach [20] is to average the microscopic equations inside porous particles over a representative elementary volume (REV), the size of which is assumed to be much larger than the characteristic size of pore structures but much smaller than the domain. Evidence from the literature indicates that a set of macroscopic equations at this scale can be derived through a rigorous volume-averaging procedure [21–23], such as the solidification process of multicomponent mixtures [24,25], the flow through the interdendritic mushy zone [26], the non-Newtonian fluid flows in porous media [27], and the flow in a stationary porous medium [28]. Based on the derived volume-averaged continuity and momentum equations, Ochoa-Tapia and Whitaker [29,30] developed certain jump conditions at the boundary between a stationary porous medium and a homogeneous fluid. To date, many efforts have been made to expand the averaging theorems, and the development of averaging equations have also been presented for porous medium systems [8,31–36]. However, it appears that a general form of the volume-averaged or macroscopic momentum equation, where the transient as well as the nonlinear inertial terms are included, has not yet been developed for a moving porous medium.

The foregoing review of the literature has prompted us to derive more general governing equations for a moving porous medium, using the volume-averaging procedure. This is the main objective of the present work. Meanwhile, in addition to the phase average velocity commonly used for porous flows [25,26,28,37], the intrinsic phase average velocity is also employed in the literature [38–40]. For example, Yang et al. [39] recently used the intrinsic phase average form of the flow velocity in the macroscopic equations, while adopted the phase average form in the flow resistance term. Similar disparity in the fluid velocity is also found in the momentum equation used by Smit et al. [40]. One direct problem resulted from such disparity is that the predicted phenomena are not pertained to the original porous media. In general, it is still not clear which kind of volume-averaged velocity should be used, especially for the case of moving porous media considered here. To our knowledge, no studies on this issue have been reported till now. This indicates the need for investigating the correct choice of flow velocity from several possible volume-averaged velocities, which is another objective of this work.

In the following, the averaging theorems regarding the time derivatives and spatial derivatives are first presented. The macroscopic equations for the incompressible flow in a moving porous medium are then derived. To solve the derived macroscopic equations, a lattice Boltzmann equation (LBE) method [41–43] is employed, and numerical simulations in two frames of reference are carried out to investigate what kind of volume-averaged fluid velocity should be chosen. The results show that Galilean invariance of the macroscopic equations can be obtained only with the intrinsic phase averaged velocity, while the use of the phase averaged velocity will break the Galilean invariance.

2. The method of volume averaging

In this work, the macroscopic governing equations for the fluid flow in a moving permeable body will be derived rigorously by averaging the microscopic continuity and momentum equations over an REV. To this end, the averaging theorems are needed to relate the average of the derivative to the derivative of the average. As shown in the literature [21–23,34,36,37,44], a number of authors developed these theorems forming the basis of the volume-averaging method. In this section, we will briefly review the invoked averaging theorems for subsequent derivations.

The flow in a moving porous medium is composed of fluid and solid phases. Assume that the fluid phase occupies a volume of V_f in a representative volume V within the porous medium. The volume occupied by the solid phase is hence $V_s = V - V_f$. In the study of multiphase transport process in porous media, the macroscopic quantities are commonly defined by volume-averaging at the REV scale. In order to obtain meaningful results, the involved three length scales should satisfy the inequality

$$l_p \ll l_r \ll l_m, \tag{1}$$

where l_p is the microscopic scale associated with the pore space, l_r is the characteristic length of the averaging volume or area, and l_m is the characteristic length of the global system under study. The length scale constraints ensure that the average quantities over the REV will be insensitive to the size of the averaging region.

The volume averaged quantities are assigned to the centroid of the REV, and three versions are in general distinguished with different definitions [37]. The first of these is the *intrinsic phase average* defined by

$$\langle\psi_k\rangle^k = \frac{1}{V_k}\int_{V_k}\psi_k dV, \tag{2}$$

where V_k represents the volume of the k-phase within the representative volume V, ψ_k is a quantity associated with the k-phase, and $k \in \{f, s\}$ with 'f' and 's' respectively denoting the fluid and solid phases. The second is the *phase average* which is the most commonly encountered averaged quantity [22,23]

$$\langle\psi_k\rangle = \frac{1}{V}\int_{V_k}\psi_k dV. \tag{3}$$

With the two definitions, one can obtain the following relation

$$\langle\psi_k\rangle = \varepsilon_k\langle\psi_k\rangle^k, \tag{4}$$

where $\varepsilon_k = V_k/V$ is the local volume fraction of the k-phase. The third average is the *spatial average* of a quantity ψ given by

$$\hat{\psi} = \frac{1}{V}\int_V \psi dV, \tag{5}$$

which assume ψ can be defined in both phases and the average is taken over the fluid and solid phases. As noted elsewhere [30,37], the spatial average appears as an unimportant variable especially in the volume-averaged equations. Thus, our discussions on the flow velocity in the macroscopic equations will focus on the other two averages as defined above and consistently in this paper.

The macroscopic conservation equations are obtained by averaging the microscopic equations over a representative volume,