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Solution of an inverse axisymmetric heat conduction problem in complicated geometry

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ABSTRACT

The aim of this work is the formulation of a space marching method, which can be used to solve inverse axisymmetric heat conduction problems in thick-walled complex shaped elements. The method has to reconstruct the transient temperature distribution in a whole construction element based on measured temperatures taken at selected points inside or on the outer surface of the construction element. The developed method is applied for temperature identification in a heated steam gate valve. The presented method is tested using the measured temperatures generated from a direct solution. Transient temperature distribution obtained from the method presented below are compared with the values obtained from the direct solution.

The presented method can be used for optimization of the power block's start-up and shut-down operations, may allow for heat loss reduction during these operations and can extend the power block's life. The presented method herein can be applied to monitoring systems that work in conventional as well as in nuclear power plants.

1. Introduction

The transient operations cause high stresses in power block devices, especially during start-up or shut-down processes. Thus, these operations should be carried out in such a manner that the total stresses do not exceed the allowable limit. Total stresses can be calculated based on measured steam pressure and based on the temperature distribution in power block device. In order to determine the transient temperature distribution in the thick-walled construction element, information regarding all the boundary conditions is needed. Typically, the outer surfaces of the elements are thermally insolated and it is much more difficult to determine the boundary condition on the inner surface. For the determination of the convective boundary condition at the inner surface the information of fluid temperature and heat transfer coefficient is necessary. Transient heat transfer coefficients can be calculated using CFD methods or from correlations found in numerous literature, however, these two methods are not very accurate especially when water vapor condenses.

An interesting method for solving this problem is to include additional temperature measurements instead of the unknown boundary condition and then to solve a transient inverse heat conduction problem (IHCP). Temperature history can be measured on the outer surface or inside the element [17]. It is very difficult to measure the temperature of the inner surface as was presented in [17,18]. In order to avoid contact between the thermoelement on the inside surface and the cold water, the thermoelement was installed in a 1 mm deep hole and covered by a metal foil which is fixed by welding it to the vessel's wall [17]. In spite of this cover and small distance from the inner surface, the temperature of the thermoelement was closer to the temperature of the water than to the inner surface temperature. The presented inverse solution in [17] gave better results. Simple algorithms based on finite difference method [1–4,17] were applied to solve one-dimensional inverse problems that occur in simple-shape bodies. The algorithms used for inverse problems in two or three dimensions are presented in [5,6]. However, they are used for elements of simple geometry. The solution of an inverse problem in complex shape bodies was possible by using the finite element control volume method. The proposition to use this algorithm for a triangular finite element can be found in [7]. An interesting method for a multiple connected region is shown in [8]. The presented algorithm solves the Poisson problem with unknown values of the source function in the cooling channels of the gas-turbine blade. Such formulation of the problem locates it in the class of inverse problems. Three-dimensional ill-posed boundary inverse problems are solved by an iterative regularization method in [19]. A general method for

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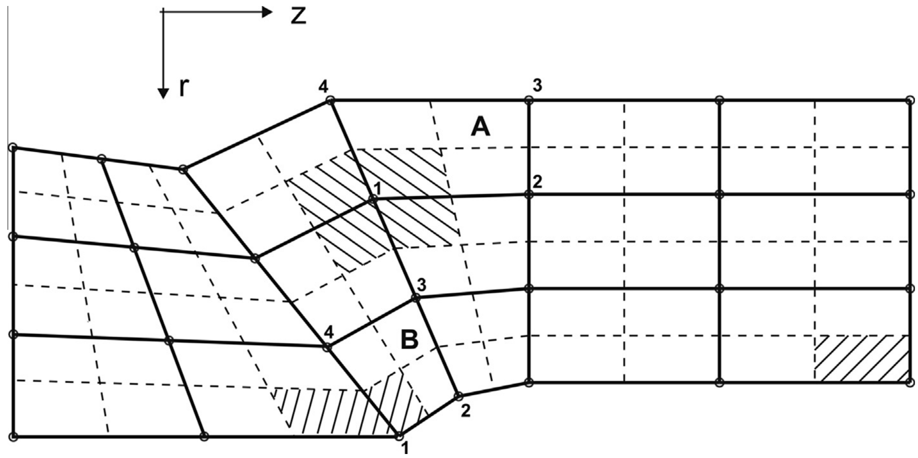

Fig. 1. Discretization of axisymmetric irregularly shaped domain.

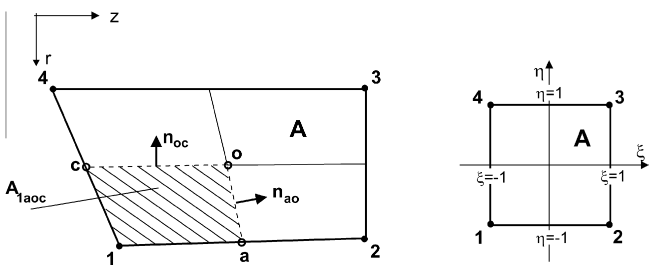

Fig. 2. Part of the control volume associated with an internal node and four node quadrilateral element in local and normalize local coordinates.

solving multidimensional inverse heat conduction is presented in [18]. Inverse problem is solved by minimization of functional defined as the sum of squares of the difference between the calculated and measured temperatures. In this formulation, the regularization term is introduced to stabilize the ill-posed problem. This idea was developed later in [20]. However, a number of problems occurred, regarding selection of temperature measurement points and the necessity to use the algorithm for the on-line monitoring of the power block devices. The advantage of the proposed space marching method is its simplicity. This simplified method can be used in the on-line mode.

The purpose of this work is to formulate a simple method, which can be used to solve nonlinear inverse heat conduction problems in thick-walled complex shaped elements in the on-line mode. The method belongs to the group of space marching methods [12]. The procedure is started at a spatial node where the temperature sensor is located and sequentially marches through space to the surface with an unknown boundary.

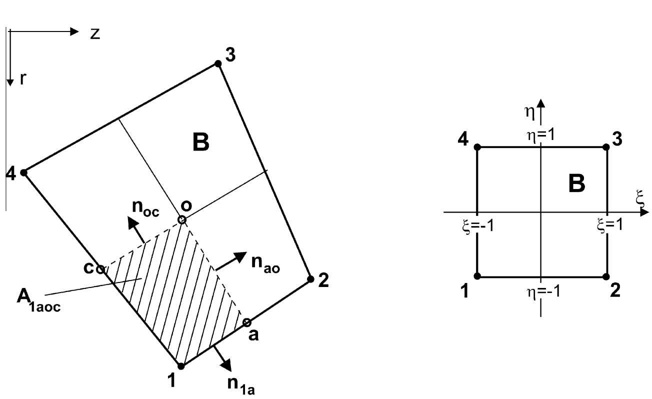

Fig. 3. Part of the control volume associated with a boundary node and a four-node quadrilateral element in local and normalize local coordinates.

2. Formulation of the problem

Three-dimensional heat conduction analysis in many components of a power unit, such as: turbine housings, T-pipes, boiler drums, headers, steam gate valves can be sometimes simplified to axisymmetric one.

If temperature in the chosen area does not vary along the circumference of the cylindrical element but it varies along the generatrix, and through the wall thickness, unsteady temperature distribution is thus axisymmetric. Cylindrical coordinate system (r, z) is introduced and transient temperature can be written as $T(r, z, t)$ [9].

The equation governing the transient heat conduction problem is given by

$$c(T)\rho(T)\frac{\partial T}{\partial t} = -\nabla\cdot\mathbf{q} + q_V, \tag{1}$$

where $\mathbf{q}$ is the heat flux vector defined by Fourier's law, q_V is the heat generation rate per unit volume,

$$\mathbf{q} = -\mathbf{D}\nabla T. \tag{2}$$

$\mathbf{D}$ is the conductivity matrix and in two-dimensional axisymmetric case it has the form

$$\mathbf{D} = \begin{bmatrix} k_r(T) & 0 \\ 0 & k_z(T) \end{bmatrix}. \tag{3}$$

For isotropic material $k_r(T) = k_z(T) = k(T)$. All material properties (c – specific heat, ρ – density, k – thermal conductivity) are assumed as known functions of temperature. The control volume finite element method is used to solve problems that occur in elements of complex geometry [10]. Eq. (1) is integrated over a general control volume V with a bounding surface S:

$$\int_V c(T)\rho(T)\frac{\partial T}{\partial t}dV = -\int_V \nabla\cdot\mathbf{q}dV + \int_V q_V dV. \tag{4}$$

The control volume V is a volume of a torus formed by rotation of a discretized surface around the z axis (Fig. 1).

By applying the mean value theorem for integrals on the left and the divergence theorem on the right, the following equation is constructed,

$$Vc(\bar{T})\rho(\bar{T})\frac{d\bar{T}}{dt} = -\int_S \mathbf{q}\cdot\mathbf{n}\,dS + \bar{q}_V V, \tag{5}$$