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Unsteady thermal boundary layer flows of a Bingham fluid in a porous medium

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ABSTRACT

In this paper we consider some unsteady free convection flows of a Bingham fluid when it saturates a porous medium. These flows are induced by suddenly raising the constant temperature of a vertical bounding surface from that of the uniform ambient value to a new constant level. As time progresses heat conducts inwards and this induces flow. We consider both a semi-infinite domain and a vertical channel of finite width. Of interest here are (i) how the presence of yield surfaces alters the classical results for Newtonian flows and (ii) the manner in which the locations of the yield surfaces change as time progresses.

1. Introduction

Bingham fluids are fluids with a yield stress but which are otherwise Newtonian, i.e. they have a linear stress–strain relationship once the yield stress is exceeded. These fluids arise in a variety of natural and industrial settings including the oil industry, agriculture and the food processing industry; see for example works by Barnes [1], Jeong [2], Maßmeyer [3], Shenoy [4] and Sochi and Blunt [5]. Other commonly-studied yield-stress fluids are Casson fluids and Herschel–Bulkley fluids, and both of these have a nonlinear stress/strain relationship post-yield.

There currently exists a fairly small body of work which considers the convection of a Bingham fluid. Many of these are concerned with convection in sidewall-heated cavities which, for a Newtonian fluid, admit flow at all nonzero values of the Rayleigh. When the cavity is filled with a Bingham fluid, flow is not possible until the Rayleigh number is sufficiently large that buoyancy forces are able to overcome the yield stress. In such contexts the critical Rayleigh number appears to be a multiple of a convective Bingham number. More detailed information may be gleaned from the papers by Hassan et al. [6], Turan et al. [7–9], Vikhansky [10] and Vola et al. [11].

More directly relevant to the present paper are the analyses of Yang and Yeh [12] and Bayazitoglu et al. [13] who studied free convection in a sidewall-heated channel. Once more, convection arises whenever the Rayleigh number is sufficiently large, but the velocity profile is characterised by having two plugs of unyielded fluid placed symmetrically about the centreline of the channel and moving in opposite directions — Karimfazli and Frigaard [14] refer to this as a five-region flow because each plug is surrounded by yielding fluid thereby giving five regions which alternate between yielding and not yielding. The studies contained in [12,13] have been extended in different directions. For example, Patel and Ingham [15] also applied a driving pressure gradient and they describe the transition from the five-region natural convection velocity profile to the more familiar three-region Poiseuille profile where the unyielded plug is in the centre of the channel. Barletta and Magyari [16] consider free convection, but one of the boundaries moves parallel with itself, a Couette flow. The free convective five-region velocity profile also undergoes a transition to a three-region profile as the velocity of the boundary increases, but now the two plugs are attached to the boundaries with yielding fluid in-between. Karimfazli and Frigaard [14] also consider free convection, but the temperatures of the bounding surfaces increase linearly with distance whilst maintaining a constant local difference. They present a complicated scenario whereby the number of regions within the velocity profile increases as the convective Bingham number decreases towards zero; the caveat is that this happens

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Nomenclature

Latin letters

$\mathcal{D}$	equal to $\ln\delta$
$F(\mathrm{Rb})$	location of the yield surface
g	gravity
G	threshold body force
K	permeability
L	length scale
p	pressure
p_x	pressure gradient in the x-direction
Q	total vertical velocity flux
Ra	Darcy–Rayleigh number
Rb	Rees–Bingham number
t	time
T	temperature (dimensional)
T_0	ambient (cold) temperature
T_1	temperature of heated surface
u	vertical Darcy velocity
x	vertical coordinate
y	horizontal coordinate
z	dummy variable

Greek letters

α	thermal diffusivity
β	coefficient of cubical expansion
δ	defined in terms of Rb in Eq. (27)
η	similarity variation
η_y	location of yield surface
θ	temperature (nondimensional)
μ	dynamic viscosity
ρ	reference density
σ	heat capacity ratio

Other symbols

$-$	dimensional quantities

only when the stratification parameter is above 7.8532, otherwise a five-region flow which is similar to that of Yang and Yeh [12] is obtained in the small convective Bingham number limit.

It is also important to mention the unsteady analysis of Kleppe and Marner [17] who consider the boundary layer flow induced by a semi-infinite uniformly hot surface; this is the Bingham-fluid analogue of the Newtonian free convective boundary layer problem that was solved by Ostrach [18]. Using an unsteady solver written for a Cartesian coordinate system, they determined the evolution with time of an impulsive change in the temperature of the bounding surface, and eventually obtained steady solutions of a one-plug, three-region type.

While undertaking the preliminary literature review for the chapter by Rees [19], which is concerned with the state-of-the-art for convection of a Bingham fluid in a porous medium, the very large number of papers which exists on yield stress fluids in general includes only a relatively very small handful on convection in porous media and all of these are on boundary layer flows. Perhaps the reason for such a void in the literature is a lack of obvious applications for the convection of a Bingham fluid in porous media, but, as is always true for yield-stress fluids, there are numerical difficulties associated with determining where the yield surface is and perhaps this has deterred research on the topic.

The present paper is part of the beginning of an effort to fill that void by considering an unsteady free convection problem. We will consider the effect on a cold saturated porous medium of raising suddenly the temperature of a vertical bounding surface to a new constant level. Unlike the paper of Kleppe and Marner [17] the present idealised configuration does not have a leading edge, which means that the temperature and velocity field may be described using a non-similar analysis, and boths fields are independent of the vertical coordinate. Heat gradually diffuses from the hot surface thereby introducing buoyancy forces, and if these forces are sufficiently large then flow will arise. In this type of problem the induced flow field may be found analytically, and it is possible to determine the locations of the yield surfaces numerically using a simple Newton–Raphson scheme. Two cases are considered, namely convection in a semi-infinite domain and convection in a channel of finite and constant thickness.

2. Governing equations

One of the earliest papers to consider the flow of a Bingham fluid in a porous medium is that of Pascal [20]. He presented a threshold gradient model based on experimental observations which is the very simplest possible model that may be used. In one dimension and for isothermal flow it may be written in the form,

$$\overline{u} = \begin{cases} -\dfrac{K}{\mu}\left[1-\dfrac{G}{|\overline{p}_{\overline{x}}|}\right]\overline{p}_{\overline{x}} & \text{when } |\overline{p}_{\overline{x}}| > G, \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

where we use G to denote the threshold gradient (or threshold body force) above which the fluid yields. We see that the fluid velocity increases linearly with the pressure gradient, $\overline{p}_{\overline{x}}$, once the threshold is exceeded. A commonly used but more complicated alternative is the Buckingham–Reiner model (see [21–23]) which is given by,

$$\overline{u} = \begin{cases} -\dfrac{K}{\mu}\left[1-\dfrac{4}{3}\left(\dfrac{G}{|\overline{p}_{\overline{x}}|}\right)+\dfrac{1}{3}\left(\dfrac{G}{|\overline{p}_{\overline{x}}|}\right)^4\right]\overline{p}_{\overline{x}} & \text{when } |\overline{p}_{\overline{x}}| > G, \\ 0 & \text{otherwise}. \end{cases} \tag{2}$$

It may be shown easily that the fluid velocity rises quadratically at first once the threshold gradient is exceeded but this eventually relaxes to a linear variation. In the present paper we will confine our attention to the threshold model.

Once buoyancy is introduced as an extra body force, the threshold model given in Eq. (1) becomes,

$$\overline{u} = \begin{cases} -\dfrac{K}{\mu}\left[1-\dfrac{G}{|\overline{p}_{\overline{x}}-\rho g\beta(T-T_0)|}\right](\overline{p}_{\overline{x}}-\rho g\beta(T-T_0)) & \text{when } |\overline{p}_{\overline{x}}-\rho g\beta(T-T_0)| > G, \\ 0 & \text{otherwise}. \end{cases} \tag{3}$$

Here we have assumed that the Boussinesq approximation applies when writing down the buoyancy term, and T_0 is the initial temperature of the porous medium. It is to be noted that we have also assumed that the threshold gradient, G, is independent of temperature, and this is equivalent to having a temperature-independent yield stress in the fluid. Exceptions to this situation include heavy oils [24] where the fluid becomes Newtonian above the so-called converting temperature.

In the above equations we have taken $\overline{x}$ to be the vertical coordinate and $\overline{u}$ to be the Darcy velocity in the same direction. In this idealised problem there will be no horizontal velocity, and therefore $\overline{v} = 0$. (If $\overline{v}$ had been nonzero then it would be necessary to alter the yield criterion in Eq. (3) to obtain a pair of momentum equations which are frame-invariant; see [19].) Therefore we may allow $\overline{u}$ to be a function only of the horizontal coordinate, $\overline{y}$,