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Fluid flow and conjugated heat transfer in arbitrarily shaped channels via single domain formulation and integral transforms

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A B S T R A C T

The present work advances a recently introduced approach based on combining the Generalized Integral Transform Technique (GITT) and a single domain reformulation strategy, aimed at providing hybrid numerical–analytical solutions to convection–diffusion problems in complex physical configurations and irregular geometries. The methodology has been previously considered in the analysis of conjugated conduction–convection heat transfer problems, simultaneously modeling the heat transfer phenomena at both the fluid streams and the channels walls, by making use of coefficients represented as space variable functions with abrupt transitions occurring at the fluid–wall interfaces. The present work is aimed at extending this methodology to deal with both fluid flow and conjugated heat transfer within arbitrarily shaped channels and complex multichannel configurations, so that the solution of a cumbersome system of coupled partial differential equations defined for each individual sub-domain of the problem is avoided, with the proposition of the single-domain formulation. The reformulated problem is integral transformed through the adoption of eigenvalue problems containing the space variable coefficients, which provide the basis of the eigenfunction expansions and are responsible for recovering the transitional behavior among the different regions in the original formulation. For demonstration purposes, an application is first considered consisting of a microchannel with an irregular cross-section shape, representing a typical channel micro-fabricated through laser ablation, in which heat and fluid flow are investigated, taking into account the conjugation with the polymeric substrate. Then, a complex configuration consisting of multiple irregularly shaped channels is more closely analyzed, in order to illustrate the flexibility and robustness of the advanced hybrid approach. In both cases, the convergence behavior of the proposed expansions is presented and critical comparisons against purely numerical approaches are provided.

1. Introduction

Along the last few decades, the research efforts towards the development of numerical solutions of conjugated heat and fluid flow formulations [1–3] has been quite remarkable. The level of scientific maturity in this area is quite evident in the quite general state-of-the-art computer codes, flexible enough to handle most engineering applications, such as [4,5]. Nevertheless, the solution of the related set of mass, momentum and energy balance equations remains an important and still growing research area due to its relevant technological implications. While the main research front has always been driven by developments on discrete computational approaches, a few research groups devoted their attention to the parallel development of hybrid numerical-analytical techniques, aimed at reducing overall computational costs and achieving more robust accuracy control schemes, which to some extent resulted in a revival of some classical analytical methods. The Generalized Integral Transform Technique (GITT) [6–9] is a typical example of such hybrid methods, which employs eigenfunction expansions to analytically express the sought potential in all but one independent variable, and numerically (or eventually

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Nomenclature

d	linear dissipation operator
K	diffusion operator coefficient
k	dimensionless thermal conductivity in problem (11)
L_x, L_y, L_z	domain length in x, y, and z directions, respectively
M	truncation order of the eigenvalue problem solution
N	truncation order of the velocity field expansions
p	pressure field
P	source term
r	radial coordinate in the exact analytical solution of a circular tube
R	radius of the circular channel
T	dimensionless potential
t	dimensionless time variable
u	velocity field component in the longitudinal direction
w	transient operator coefficient
x,y	transversal coordinates
z	longitudinal coordinate

Greek letters

Δp	pressure difference between outlet and inlet
α, β	boundary conditions coefficients
ϕ	source term in boundary conditions
μ, η	eigenvalues corresponding to eigenfunctions ψ and χ, respectively
ν	kinematic viscosity
ρ	density
ψ, χ	eigenfunctions

Subscripts and superscripts

–	integral transformation
~	normalized eigenfunction
f	quantity corresponding to the fluid regions
i, j	order of eigenquantities
s	quantity corresponding to the solid regions

analytically) solve the transformed ordinary (or partial) differential system in the remaining independent variable (or variables).

The application of the integral transform approach to either conjugated heat transfer problems with wall lumped models [10–13] or internal forced convection problems in regularly shaped channels [14–17] is well documented in a number of contributions along the last two decades, mostly addressed to benchmarking and verification in the realm of classical test problems, profiting of the method's intrinsic characteristic of finding solutions with automatic global error control. In addition, the GITT approach has been advanced, since the early phases of its development, in handling diffusion problems within irregular domains, including fully developed and developing flow within irregularly shaped channels [18–24], by considering integral transformations with space dependent eigenvalues.

Convective heat transfer within irregularly shaped ducts is found in several practical applications, for instance, in the flow of liquids in chemical processing plants, air flow in cooling, heating and ventilation units, and cooling of electronic equipment. In the context of microfluidics and microsystems, besides the cooling of electronic components, one may also recall the crucial importance of convection within microchannels in various applications that require micro-heat exchangers, micro-heat spreaders, and micro-reactors [25,26]. In some of these situations the flow in irregular geometries must be tackled, either because of the complex geometries involved, or as a result of micro-fabrication irregularities. In fact, for the conception and design of thermal microsystems, several works have been devoted to building and revisiting reliable models and solution methodologies capable of describing the physical phenomena that take place in such microscale heat and fluid flow problems [27,28].

Motivated by the theoretical discussion in [29], the experimental and theoretical results in [13] have shown the importance of considering heat conduction along microchannels walls in certain situations, leading to a conjugated conduction–internal convection heat transfer problem, in this case handled through integral transformation with an improved wall lumping procedure, which yielded results in better agreement with the reported experimental data. More recently, the reformulation of conjugated conduction–convection problems as a single region model has been proposed [30], accounting for the local heat transfer at both the fluid flow and the channel wall regions, by making use of coefficients represented as space-variable functions, with abrupt transitions occurring at the fluid-solid interfaces. In this single region formulation, the mathematical model incorporates the information concerning the multiple original domains of the problem. The excellent agreement between the proposed methodology and the exact solution for a fairly simple test case presented in [30], motivated the extension of the approach, which has been demonstrated to be extendable to the solution of more involved problems, for example when dealing with axial conduction both at the channel walls and in the fluid stream [31], when taking into account the heat conduction to the upstream region of the heat exchange section [32], which may be important in problems with low Péclet numbers, in fully three dimensional conjugated heat transfer problems [33], and in a real application of multiple circular microchannels etched onto a nanocomposite substrate [34].

In these recent contributions [30–34] on conjugated heat transfer analysis, it is always considered that the duct has a regular shape such as parallel plates, circular tubes or rectangular channels, and that the fully developed velocity field is known from available analytical expressions. However, this novel approach is flexible enough to deal with arbitrarily shaped channels, even in a more direct way than previously proposed in the solution of diffusion problems through integral transforms [18,21]. Thus, in the present work we advance the combined integral transforms and single domain formulation approach to solve the flow and energy equations for conjugated heat transfer within a channel with arbitrarily shaped cross-section and its associated solid substrate. Hence, both the fully developed velocity field and temperature distributions in the fluid and the solid, are represented as eigenfunction expansions. The idea is to represent the complex domain as a simpler single domain problem with spatially variable coefficients, which account for the transitions between the original problem regions. Following the solution methodology developed in previous works [30–34], the GITT is then employed in the direct integral transformation of the problem with space-variable coefficients, as well as on the solution of the associated eigenvalue problem [35,36], in order to transform the original differential eigenvalue problems into algebraic eigensystems, to be readily solved through available built-in routines, such as in the *Mathematica* system [37].

In order to verify the adequacy of the proposed approach, first a test case is investigated, which consists of the flow inside a channel with an arbitrary cross-section shape, inspired by an actual situation of micro-fabrication through laser ablation. In addition, a more complex multichannel configuration is considered more closely,