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Accelerative iteration for coupled conductive–radiative heat transfer computation in semitransparent media

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ABSTRACT

Due to the intensive coupling effect and non-linear characteristic, coupled conductive–radiative heat transfer problem in semitransparent media always consumes an enormous amount of computing time. In present work, an accelerative iteration model is developed to speed up the convergence process of this kind of coupled calculation. It promotes the iterative efficiency by revising the temperature and incident radiation of media during the iteration of energy equation and radiative transfer equation. It can remarkably reduce the computing time of coupled conductive–radiative heat transfer calculation in semitransparent media, even in the cases with complicate boundary conditions. The accuracy of present model is verified by comparing with literature and traditional source iteration method. The influences of conduction–radiation parameter, optical thickness, scattering albedo, emissivity of walls and boundary conditions are investigated as well. The results indicate that the accuracy of present model is reliable and its accelerative effect is more significant for the optically thick and scattering-dominated media with complicate boundary conditions.

1. Introduction

In many engineering applications such as insulating material [1], plastic manufacture [2], fabrication of optical fiber [3], evaporation of liquid droplet [4], and so on, conductive–radiative heat transfer problems in semitransparent media have received more and more attention. In the last three decades, many numerical methods have been developed to solve this kind of problems. Most of previous studies were focused on the improvement of numerical accuracy or development of new methods. The studies on computational efficiency are still relative few in the community of heat transfer. Moreover, when the optimization of heat transfer is considered, lots of forward iteration computation is required. However, due to the poor computational efficiency, the optimization involved the coupled conductive–radiative problem always takes too much time. In addition, for some practical process such as evaporation of semitransparent liquid droplets [4–8], when the alternative physical state, complicate boundary conditions and temperature dependent radiative properties of semitransparent media are considered, accurate and efficient numerical method becomes a real challenge because of the intensive coupling effect and non-linear characteristic in solving such problem. Therefore, to study computational efficiency of coupled conductive–radiative problem is very necessary.

In early studies, Siewert [9] developed a Newton iteration model to improve the convergency of coupled conductive–radiative heat transfer problem. In his study, the spherical-harmonics method is used to calculate radiative transfer in semitransparent media. In addition, a fast multilevel algorithm is also developed to solve conductive–radiative heat transfer problem by Banoczi and Kelley [10]. Moreover, Mazumder [11] proposed a simultaneous solving model of temperature and incident radiation to accelerate the iteration of energy equation and radiative transfer equation, but the accuracy of this model is limited because P_1 model was employed to calculate the radiation. Miliauskas et al. [4,12] developed an integral equation method to analyze the effect of combined radiation and conduction on the evaporation of liquid droplet. Their studies show that radiation has an important influence to evaporation of liquid droplet, and a complex boundary condition must be considered in the analysis. Recently, several numerical methods based on discrete ordinary method (DOM) such as finite volume method (FVM) [13], finite element method (FEM) [14], meshless method [15] and so on, are developed to treat the coupled conductive–radiative heat transfer, and are becoming more and more popular due to their comprehensive adaptation and acceptable accuracy. Generally, these methods employ an ordinal iteration model in which the radiative transfer equation (RTE)

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Nomenclature

D_k	diffusion coefficient of Eq. (14)
G	incident radiation, $G = \int_{\Omega=4\pi} \mathbf{I} d\Omega$, $W \cdot m^{-2}$
$\mathbf{I}$	radiative intensity, $W/(m^2 \cdot sr)$
J_w	radiosity on the boundary, $W \cdot m^{-2}$
M	number of discrete directions
N	conduction–radiation parameter, $N = k\beta/4\sigma T_{ref}^3$
T	temperature, K
W	weighting function of finite element
h	convective heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
k	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
$\mathbf{n}$	unit normal vector of boundary surface
$\mathbf{q_r}$	radiative heat flux, $W \cdot m^{-2}$
$\mathbf{r}$	location
s	relaxation factor
v	wavelength, m
w_m	solid angular weight
Φ	scattering phase function
Γ	boundary condition
α	rectification value of temperature
ε	emissivity
γ_e, γ_s	rectification value of incident radiation for emissive and scattering item
κ_a	absorption coefficient, m^{-1}
σ	Stefan–Boltzmann constant, $5.67 \times 10^{-8}\ W \cdot m^{-2} \cdot K^{-4}$
σ_s	scattering coefficient, m^{-1}
ω	scattering albedo
ξ_I, ξ_T	convergence criterion of radiative transfer equation and energy equation
τ	optical thickness
Subscript	
b	blackbody
m	indices for ordinate directions
ref	reference value
w	wall
$*$	accuracy value
Superscript	
n	iteration times

and energy equation (EE) is solved successively. However, when a scattering-dominated media or optically thick media is considered, such iteration is very inefficient and converges slowly [16]. Moreover, when a complicate boundary condition is considered, this deficiency is very obvious. Therefore, these numerical models based on DOM should be revised further to improve their iterative efficiency.

In present work, the solving procedure of coupled conductive–radiative problem is analyzed firstly. Based on such analysis, an accelerative iteration model is developed to improve the convergence. FEM is employed to realize the present accelerative iteration model and the detailed mathematical formula and algorithm are discussed. The accuracy of present model is verified by comparing the results with those from the literature and FEM without any acceleration technique. The effects of the conduction–radiation parameter, scattering albedo, emissivity of walls, optical thickness and boundary conditions on the performance of present acceleration model are investigated as well.

2. Mathematical formulation

The energy equation of stable conductive–radiative heat transfer in the Cartesian coordinate system can be written as [17]:

$$k\nabla^2 T - \int_0^\infty \kappa_a(v)[4\pi I_b(v) - G(v)]dv = 0 \tag{1}$$

For a grey media, the nonlinear T^4 item derived from the integral item of wavelength, will present in above equation. In numerical models, the nonlinear item must be reduced to a linear function of temperature. Generally, it can be approximated by the Taylor formation. Then, eq. (1) can be written as:

$$-k\nabla^2 T_n + (16\sigma T_{n-1}^3 \kappa_a) \cdot T_n = \kappa_a G_{n-1} + 12\sigma T_{n-1}^4 \kappa_a \tag{2}$$

where, the subscript n is the number of iteration times, the incident radiation G is defined as following equation:

$$G = \int_{\Omega=4\pi} \mathbf{I} d\Omega \tag{3}$$

The boundary conditions Γ of Eq. (2) can be written as:

$$T = T_w \qquad \text{when } \Gamma \in \Gamma_1 \tag{4-a}$$

$$-k\nabla \cdot T + \mathbf{q_r} \cdot \mathbf{n} = q_w \qquad \text{when } \Gamma \in \Gamma_2 \tag{4-b}$$

$$-k\nabla \cdot T + \mathbf{q_r} \cdot \mathbf{n} = h_w(T - T_f) \qquad \text{when } \Gamma \in \Gamma_3 \tag{4-c}$$

where the radiative heat flux on the boundary $\mathbf{q_r} \cdot \mathbf{n}$ can be written as:

$$\mathbf{q_r} \cdot \mathbf{n} = \int_{\mathbf{n}\cdot\Omega<0} \mathbf{I}|\mathbf{n} \cdot \Omega| d\Omega \tag{5}$$

$\mathbf{n}$ is the inner normal vector on the boundary, $|\mathbf{n} \cdot \Omega|$ represents the inner product of the Ω direction and $\mathbf{n}$. The κ_a is the absorption coefficient of media. Γ_1, Γ_2 and Γ_3 represents the fixed-temperature boundary condition, fixed-heat-flux boundary condition and the convective boundary condition, respectively. h_w is the convective heat transfer coefficient. T_f is the temperature of outer fluid. It can be found from Eqs. (2–5) that the energy equation and its boundary condition contain the unknown variable $\mathbf{I}(\mathbf{r}, \mathbf{\Omega})$ which is the function of location and direction. When considering the gray and diffuse reflection boundary, the radiative intensity $\mathbf{I}(\mathbf{r}, \mathbf{\Omega})$ of semitransparent media can be obtained by solving the following radiative transfer equation (RTE) [18].

$$(\mathbf{\Omega} \cdot \nabla)\mathbf{I}(\mathbf{r}, \mathbf{\Omega}) = -(\kappa_a + \sigma_s)\mathbf{I}(\mathbf{r}, \mathbf{\Omega}) + \frac{\kappa_a}{\pi}\sigma T^4 + \frac{\sigma_s}{4\pi} \times \int_{\Omega'=4\pi} \mathbf{I}(\mathbf{r}, \mathbf{\Omega}')\Phi(\mathbf{\Omega}', \mathbf{\Omega})d\mathbf{\Omega}' \tag{6}$$

The boundary condition of RTE can be written as follows:

$$\mathbf{I}(\mathbf{r}_w, \mathbf{\Omega}) = \frac{\varepsilon_w}{\pi}\sigma T^4 + \frac{1-\varepsilon_w}{\pi}\int_{\mathbf{n}\cdot\Omega'<0} \mathbf{I}(\mathbf{r}_w, \mathbf{\Omega}')|\mathbf{n} \cdot \mathbf{\Omega}'|d\mathbf{\Omega}' \quad (\mathbf{n} \cdot \mathbf{\Omega} > 0) \tag{7}$$

where ε_w is the wall emissivity. It can be found that the right side of Eq. (6) contains the temperature resolved emissive item $\frac{\kappa_a}{\pi}\sigma T^4$, and its boundary condition also has an emissive item $\frac{\varepsilon_w}{\pi}\sigma T^4$. Therefore, energy equation and RTE are coupled with each other, and must be solved by an iteration model.