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Turbulent free convection in a porous cavity using the two-temperature model and the high Reynolds closure

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ABSTRACT

This work presents a study on free convection in a porous square cavity saturated with a Newtonian fluid. Computations for laminar and turbulent flow are performed. Governing equations were time- and volume averaged according to the double-decomposition concept. Discretization of governing equations was obtained with the control-volume approach and the system of algebraic equation was relaxed via the SIMPLE method. Two energy models were employed, namely the one- and two-temperature models. Results indicated that when the ratio of thermal conductivities equals unity, both models give similar results. However, the overall Nusselt number across the cavity is reduced as porosity or the thermal conductivity ratio increases. A critical value for the Rayleigh number, understood as that when laminar and turbulent solution differ by a substantial amount, was found to be a function of the thermal conductivity ratio.

1. Introduction

The study of heat transport across a porous cavity is already widespread in the literature and there are many industrial applications for such analyses. However, most of the existing results are concerned with laminar flows, which are an exception in the industrial environment. In the majority of engineering flows of practical relevance turbulent regime prevails and, as such, development of more accurate models for turbulent free convection in porous media can benefit the design and analysis of more efficient engineering equipment.

Many studies have been published in the literature about laminar flow in porous media: The monographs of Nield and Bejan [1] and Ingham and Pop [2] fully documented the problem of a laminar flow in a porous medium. The works of Walker and Homsy [3], Bejan [4], Prasad and Kulacki [5], Beckermann et al. [6], Gross et al. [7], Manole and Lage [8] and Moya et al. [9] have contributed with some important results to the problem of natural convection in a porous rectangular cavity. The work of Baytas and Pop [10], concerned a numerical study of the steady free convection flow in rectangular and oblique cavities filled with homogeneous porous media using a nonlinear axis transformation.

Usually, modeling of macroscopic transport for incompressible flows in rigid porous media has been based on the volume-average methodology for either heat or mass transfer [11–14]. If fluctuations in time are also of concern due the existence of turbulence in the intra-pore space, a variety of mathematical models have been published in the literature in the last decade. One of such views, which entails simultaneous application of both time and volume averaging operators to all governing equations, has been organized and published in a book [15] that describes, in detail, an idea known in the literature as the double-decomposition concept (see Section 3, pgs. 27–32 in [15] for details). Paramount contributions to turbulence modeling in porous media using different approaches have been also published in the open literature [16,17].

Extension of the double-decomposition theory of [15] for treating turbulent natural convection in cavities using thermal equilibrium [18] as well as a two temperature model [19] has also been documented. In the literature, the use of the two-energy equation model has also been considered for passive heat transfer across differentially heated cavities [20].

Recently, Carvalho and de Lemos [21] studied turbulent free convection in square porous cavity using the thermal equilibrium model, or say, an average temperature was assumed to represent both the fluid and the solid porous matrix. A number of analyses were presented in [21] that were not included in [18], broaden, as such, studies on passive turbulent heat transfer using the one-energy equation model. Later, Carvalho and de Lemos [22] used the two-energy equation model for analyzing laminar flows in cavities. Therein, only laminar regime was investigated.

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Nomenclature

Latin characters

c_F	Forchheimer coefficient
cp	specific heat
d	pore diameter
$\mathbf{D}$	$\mathbf{D} = [\nabla\mathbf{u} + (\nabla\mathbf{u})^T]/2$, deformation rate tensor
Da	Darcy number, $Da = \frac{K}{H^2}$
D	$\sqrt{\frac{144\,K(1-\phi)^2}{\phi^3}}$, particle diameter
$\mathbf{g}$	gravity acceleration vector
G^i	generation rate of $\langle k\rangle^i$ due to the action of the porous matrix
G^i_β	generation rate of $\langle k\rangle^i$ due to the buoyant effects
h	heat transfer coefficient
H	square height
$\mathbf{I}$	unit tensor
K	permeability
k_f	fluid thermal conductivity
k_s	solid thermal conductivity
$\mathbf{K}_{disp}$	conductivity tensor due to the dispersion
$\mathbf{K}_{tor}$	conductivity tensor due to the tortuosity
L	cavity width
$\mathbf{n}_i$	unit vector normal to the interfacial area A_i
Nu	$Nu = hL/K_{eff}$,Nusselt number
Nu_w	average Nusselt number at hot wall
Pr	Prandtl number
Ra_f	$Ra_f = \frac{g\beta H^3 \Delta T}{v_f \alpha_f}$, fluid Rayleigh number
Ra_m	$Ra_m = Ra_f \cdot Da = \frac{g\beta_\phi H \Delta T K}{v_f \alpha_{eff}}$, Darcy–Rayleigh number
Ra_{cr}	critical Rayleigh number
Re_D	$Re_D = \frac{\lvert\bar{\mathbf{u}}_D\rvert D}{v_f}$, Reynolds number based on the particle diameter.
T	temperature
$\mathbf{u}$	microscopic velocity
$\mathbf{u}_D$	Darcy or superficial velocity (volume average of $\mathbf{u}$)

Greek characters

α	thermal diffusivity
β	thermal expansion coefficient
ΔV	representative elementary volume
ΔV_f	fluid volume inside ΔV
ε	$\varepsilon = \mu\overline{\nabla\mathbf{u}' : (\nabla\mathbf{u}')^T}/\rho$, dissipation rate of turbulent kinetic energy.
μ	dynamic viscosity
v	kinematic viscosity
ρ	density
σ's	non-dimensional constants
ϕ	$\phi = \Delta V_f/\Delta V$, porosity

Special characters

φ	general variable
$\bar{\varphi}$	time average
φ'	time fluctuation
$\langle\varphi\rangle^i$	intrinsic average
$\langle\varphi\rangle^v$	volume average
$^i\varphi$	spatial deviation
$\lvert\boldsymbol{\varphi}\rvert$	absolute value (Abs)
$\boldsymbol{\varphi}$	general vector variable
φ_{eff}	effective value, $\varphi_{eff} = \phi\varphi_f + (1-\phi)\varphi_s$
$\varphi_{s,f}$	solid/fluid
$\varphi_{H,C}$	hot/cold
φ_ϕ	macroscopic value
$()^T$	transpose

Therefore, this purpose of this work is to extend the simulations of Carvalho and de Lemos [21] considering now the thermal non-equilibrium hypothesis of [22]. By extending the two-energy equation model to simulations of turbulent flows in porous cavities a wide variety of important engineering systems of practical interest can be modeled in a more realistic fashion.

2. Governing equations

The mathematical models here employed are based in the work of [18] including now the assumption of Local Thermal Non-Equilibrium (LTNE) or Two-temperature equation model for heat transfer calculations [22]. As most of the theoretical development is readily available in the open literature, the governing equations will be just presented and details about their derivations can be obtained in the above-mentioned papers. Essentially, local instantaneous equations are volume-averaged using appropriate mathematical tools [23].

2.1. Macroscopic continuity equation

The macroscopic equation of continuity for an incompressible fluid flowing through a porous medium is given by:

$$\nabla \cdot \bar{\mathbf{u}}_D = 0 \tag{1}$$

where $\bar{\mathbf{u}}_D$ is the time averaged surface velocity, also known as Darcy velocity. In Eq. (1) the Dupuit–Forchheimer relationship, $\bar{\mathbf{u}}_D = \phi\langle\bar{\mathbf{u}}\rangle^i$, has been used where ϕ is the porous medium porosity and $\langle\bar{\mathbf{u}}\rangle^i$ identifies the intrinsic (fluid phase) average of the local velocity vector $\mathbf{u}$ [23].

2.2. Macroscopic momentum equation

The macroscopic momentum equation (Navier–Stokes) for an incompressible fluid with constant properties flowing through a porous medium can be written as:

$$\rho\left[\frac{\partial\bar{\mathbf{u}}_D}{\partial t} + \nabla\cdot\left(\frac{\bar{\mathbf{u}}_D\bar{\mathbf{u}}_D}{\phi}\right)\right] = -\nabla\left(\phi\langle\bar{p}\rangle^i\right) + \mu\nabla^2\bar{\mathbf{u}}_D + \nabla\cdot\left(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right) - \rho\beta_\phi\mathbf{g}\phi\left(\langle\bar{T}_f\rangle^i - T_{ref}\right) - \left[\frac{\mu\phi}{K}\bar{\mathbf{u}}_D + \frac{c_F\phi\rho\lvert\bar{\mathbf{u}}_D\rvert\bar{\mathbf{u}}_D}{\sqrt{K}}\right] \tag{2}$$

where the last two terms in Eq. (2) represent the Darcy and the Forchheimer terms, respectively. The symbol K is the porous medium permeability, $c_F = 0.55$ is the form drag coefficient, $\langle\bar{p}\rangle^i$ is the intrinsic average pressure of the fluid, ρ is the fluid density and μ represents the fluid viscosity. Gravity acceleration is defined by $\mathbf{g}$ and β_ϕ is the macroscopic thermal expansion coefficient defined as $\beta_\phi = \frac{\langle\rho\beta(\bar{T}_f - T_{ref})\rangle^v}{\rho\phi(\langle\bar{T}_f\rangle^i - T_{ref})}$ [18]. The term $-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$ is known as the Macroscopic Reynolds Stress Tensor (MRST) and is given by:

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i = \mu_{t_\phi}2\langle\bar{\mathbf{D}}\rangle^{\mathbf{v}} - \frac{2}{3}\phi\rho\langle k\rangle^i\mathbf{I} \tag{3}$$

where

$$\langle\bar{\mathbf{D}}\rangle^v = \frac{1}{2}\left\{\nabla(\phi\langle\bar{\mathbf{u}}\rangle^i) + [\nabla(\phi\langle\bar{\mathbf{u}}\rangle^i)]^T\right\} \tag{4}$$