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Conjugate natural convection in a square porous cavity filled by a nanofluid using Buongiorno's mathematical model

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ABSTRACT

Steady-state natural convection heat transfer in a square porous enclosure having solid walls of finite thickness and conductivity filled by a nanofluid using the mathematical nanofluid model proposed by Buongiorno is presented. The nanofluid model takes into account the Brownian diffusion and thermophoresis effects. The study is formulated in terms of the vorticity-stream function procedure. The governing equations were solved by finite difference method and solution of algebraic equations was made on the basis of successive under relaxation method. Effort has been focused on the effects of seven types of influential factors such as the Rayleigh and Lewis numbers, the buoyancy-ratio parameter, the Brownian motion parameter, the thermophoresis parameter, the thermal conductivity ratio, and solid walls thickness on the fluid flow and heat transfer. Streamlines, isotherms, isoconcentrations, local Nusselt and Sherwood numbers are presented. It has been found that the local Nusselt number at the solid-porous interface ($x = D$) is an increasing function of *Ra*, *Nr* and a decreasing function of *Nt*, *Le* and *D*. An effect of K_r on *Nu* and *Sh* is non-monotonic. Ranges of key parameters for which a non-homogeneous model is more appropriate for the description of the system have been determined.

1. Introduction

Natural convection heat transfer in a porous medium occurs in nature, science and engineering which are important from theoretical as well as practical point of view and has been an area of intensive investigation for the last several decades. This is due to the fact that a model of convection fluid within a porous matrix has a wide range of applications spanning from a transport process in a biomechanical system, such as blood in the pulmonary alveolar sheet, to a large scale circulation of brine in a geothermal reservoir, etc. The monograph by Nield and Bejan [1] serves as an excellent review on what has been achieved in this field up to now. Heating and cooling are important in many industrial and engineering applications, such as aerodynamic extrusion of plastic sheet, the cooling of metallic plate in a cooling bath, and in thin film solar energy collector device, solar receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchanges placed in a low-velocity environment, flows in the ocean and in the atmosphere, etc. A comprehensive review of buoyancy induced flows of viscous fluids is given in the books by Gebhart et al. [2], Schlichting and Gersten [3], and Pop and Ingham [4].

In many problem formulations of natural convection a heating condition along the boundary that forms the solid–fluid interface is usually prescribed. The condition of either a constant temperature or a constant heat flux is commonly used. There is, however, the question as to what surface boundary condition has to be applied on the surface temperature. Despite the routinely employed condition of either a constant temperature or a constant heat flux is commonly used, it arrises in practice only when the solid body has a much greater thermal conductivity compared to the effective one of the porous medium. This is equivalent to saying that the Biot number of the problem is not zero, but finite. In real applications, therefore, the Biot number has a finite value and the thermal boundary conditions can rarely be known in advance. Neither a prescribed temperature or heat flux would seem entirely appropriate, a convective condition, being in essence a combination of these two conditions, is also realisable in practice and it should provide further useful insights. However, in many studies of convective flow in porous cavities, the walls of the cavities are assumed to be zero thickness and conduction in the walls

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Nomenclature

C	nanoparticle volume fraction
C_0	reference value of nanoparticle volume fraction
d	dimensional thickness of the heat-conducting solid walls
D	dimensionless thickness of the heat-conducting solid walls
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
g	gravitational acceleration
k_1	thermal conductivity of solid walls material
k_2	thermal conductivity of porous medium
$\bar{K}$	permeability of the porous medium
K_r	thermal conductivity ratio
L	square cavity size
Le	Lewis number
$m_{w,0}$	mass flux from the left solid-porous interface
Nb	Brownian motion parameter
Nr	buoyancy-ratio parameter
Nt	thermophoresis parameter
Nu	local Nusselt number
$\overline{Nu}$	mean Nusselt number
$q_{w,0}$	heat flux from the left solid-porous interface
Ra	Rayleigh number for the porous medium
Sh	local Sherwood number
$\overline{Sh}$	mean Sherwood number
T	temperature of the nanofluid
T_c	temperature of the cooled wall
T_h	temperature of the hot wall
$\bar{u}, \bar{v}$	dimensional velocity components along the axes $\bar{x}$ and $\bar{y}$
$\bar{x}$	dimensional coordinate measured along the bottom wall of the cavity
$\bar{y}$	dimensional coordinate measured along the vertical wall
x, y	dimensionless Cartesian coordinates

Greek letters

α_m	effective thermal diffusivity of the porous medium
β	volumetric expansion coefficient of the fluid
ε	porosity of the porous medium
ϕ	rescaled nanoparticle volume fraction
μ	dynamic viscosity
θ	dimensionless temperature
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho C_p)_f$	heat capacity of the base fluid
$(\rho C_p)_p$	effective heat capacity of the nanoparticle material
τ	parameter defined by $\tau = (\rho C_p)_f/(\rho C_p)_p$
$\bar{\psi}$	dimensional stream function
ψ	dimensionless stream function

can have an important effect on the flow and heat transfer characteristics (see Lauriat [5], Saeid [6,7], Aleshkova and Sheremet [8], Sheremet and Trifonova [9]). Basic assumptions underlying mathematical models of conduction–convection conjugate heat transfer process in porous media is described in the review paper by Kimura et al. [10].

In recent years, the study on nanofluids has become a hot topic among researchers because in the presence of the nanoparticles in the fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics. The term *nanofluid* represents the fluid in which nano-scale particles are suspended in the base fluid with low thermal conductivity such as water, ethylene glycol (EG), oils, etc. Buongiorno [11] developed a non-homogeneous equilibrium model by considering the effect of the Brownian diffusion and thermophoresis, that are important slip mechanisms in nanofluid. This model has been widely used to study the free convection boundary layer flow past a vertical flat plate embedded in a porous medium filled by nanofluids [12–15].

Although there are many papers on conjugate convective heat transfer in cavities filled with nanofluids [16–20], there are, by our best of knowledge, no studies that considered the conjugate natural convection in porous cavities filled by a nanofluid using the mathematical nanofluid model proposed by Buongiorno [11]. The physical situation described in papers [12–15] shows that the work on porous media filled by nanofluids is not just a mathematical exercise, but is based on deep physical understanding of nanofluid flows. This demonstrates that we are simulating here a real physical problem of natural convection in a square porous cavity filled by a nanofluid. The objective of the present paper is, therefore, to numerically investigate the steady conjugate natural convection heat transfer within a square porous cavity filled by a nanofluid with vertical heat-conducting solid walls of finite thickness using the mathematical nanofluid model proposed by Buongiorno [11] in combination with Darcy's law for the flow in the porous medium and the Boussinesq approximation for the buoyancy force. To the best of our knowledge, this problem is new and has not been studied before.

2. Basic equations

Consider the natural convection in a square porous cavity with heat-conducting solid walls of finite thickness d filled with a nanofluid based on water and nanoparticles. A schematic geometry of the problem under investigation is shown in Fig. 1, where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates and L is the square cavity size. It is assumed that the vertical left wall ($\bar{x} = 0$) is maintain at temperature T_h, while the right wall ($\bar{x} = L$) is kept at a temperature T_c, where we assume that $T_h > T_c$. It is also assumed that the

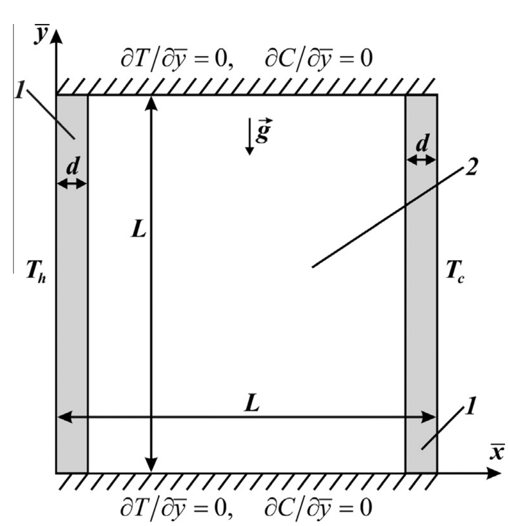

Fig. 1. Physical model and coordinate system, where 1 – solid walls and 2 – porous cavity.