International Journal of Heat and Mass Transfer 79 (2014) 291–299

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Constructal design of thermoelectric power packages

Adrian Bejan [a,*], Sylvie Lorente [b], Deok-Hong Kang [c]

[a] Department of Mechanical Engineering and Materials Science, Duke University, Durham, NC 27708-0300, USA
[b] Université de Toulouse, INSA, 135 Avenue de Rangueil, 31077 Toulouse, France
[c] Research Institute of Industrial Science and Technology, Energy & Resources Research Department, Pohang, Republic of Korea

ARTICLE INFO

Article history:
Received 11 July 2014
Received in revised form 4 August 2014
Accepted 5 August 2014

Keywords:
Thermoelectric
Constructal
Vascular
Serpentine

ABSTRACT

In this paper we consider the complete thermofluid design and performance of a thermoelectric module. We increase the temperature difference that must be maintained across the module, and at the same time we reduce the pumping power required by the streams that bathe the hot and cold plates of the module. We find that for greater power output the two streams must be configured in parallel, not in counterflow, and not between two well mixed plenums. We also find that the loss of thermoelectric power due to the temperature nonuniformity of the two plates competes with the power lost during the pumping of the two streams, and that from this competition results the optimal mass flow rates of the two streams. At the optimum, the maximized power output of the module is proportional to a group of geometric parameters (Eq. (39)), which can be maximized further by designing vascular flow architectures for the two plates. The vascular designs reveal an optimal ratio of channel diameters, optimal plate aspect ratio, and a channel flow volume fraction that is of the same order as the ratio of the fluid thermal conductivity divided by the solid thermal conductivity. The flow architectures are further illustrated with numerical examples of vascular and serpentine configuration and performance.

1. Thermoelectric power generation

Thermoelectric power generation (TE) is attracting interest from across the spectrum of technology because it offers a path to clean, renewable energy. In addition, TE devices offer compactness and simplicity because they do not have moving parts. The efficiency of TE designs is limited by the transport properties of the thermoelectric materials accounted for in the property group Z discussed in Eq. (1). Progress toward greater efficiencies can be made by optimizing the geometry—the relative shapes and sizes—of the legs of the module [1–3]. This and other aspects of the optimization of thermoelectric modules (TEM) has generated a growing body of literature [1–19].

In the present study we consider the whole picture—the thermofluid design and performance of the TEM. We consider not only the temperature difference that must be maintained across the TEM, but also the fluid mechanics of the vascular hot and cold plates that sandwich the TEM. We show that important tradeoffs exist in the thermofluid design such that the overall performance of the TEM is increased.

The thermoelectric module (TEM) is defined in Fig. 1. The hot plate is heated by a stream of hot oil. The cold plate is cooled by a stream of water. Heat flows by conduction perpendicular to the plates, from the hot plate to the cold plate.

Larger stacks are obtained by sandwiching several TEMs as shown in Fig. 2. All the TEMs are identical. Each is a sandwich of three parts: hot plate (H), thermoelectric converter (T), and cold plate (C), in other words, HTC, or CTH. There are two ways to assemble such elements in a stack:

(a) Alternating orientations, i.e., plates of the same temperature touching each other:

HTC CTH HTC...

(b) Elements oriented in the same way:

HTC I HTC I HTC...

The simplest design is (a). In design (b), a layer of insulation (I) must be placed between the C and H plates of adjacent elements. Fig. 2 shows the (a) rule of assembly.

If the TEM is small enough and the hot and cold fluid flow rates are large enough, then the hot and cold plates are essentially isothermal, at T_H and T_L, respectively. The thermodynamics of the TEM is detailed in Ref. [1], and it shows that the maximum thermodynamic efficiency of a module with isothermal ends is

---

[*] Corresponding author.
E-mail address: abejan@duke.edu (A. Bejan).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.08.008
0017-9310/

Nomenclature

A	plate area, m^2
C	factor, dimensionless
c_P	specific heat at constant pressure, J kg^{-1} K^{-1}
d	spacing between channels, m
D_2	channel thickness scale, m
D_1	trunk thickness, m
D_2	branch thickness, m
G	function of geometry, Eq. (39)
H	transversal dimension of area, m
I	integral
k	effective TEM thermal conductivity, W m^{-1} K^{-1}
k_f	fluid thermal conductivity, W m^{-1} K^{-1}
k_s	solid plate conductivity, W m^{-1} K^{-1}
$K_{1,2}$	factors, Eqs. (37) and (38)
L	transverse dimension of area, m
L_s	length scale, m
m	number of channels (branches)
$\dot{m}$	mass flow rate, kg s^{-1}
n	number of channels (branches)
N	number of heat transfer units, Eq. (11)
p	perimeter of contact, m
P	pressure, Pa
q_H	heat current, W
R	electric resistance, Ω
R_{th}	thermal resistance, K W^{-1}
Sv	svelteness number
t	thickness of TEM, m
t	spacing between channels, m
T_H	high temperature, K
T_L	low temperature, K
TEM	thermoelectric module
U	overall heat transfer coefficient, W m^{-2} K^{-1}
V	volume, m^3
W	power, W
W_{lost}	lost power, W
W_{max}	maximum power, W
$W_{\Delta P}$	pumping power, W
$W_{\Delta T}$	power lost due to temperature nonuniformity, W
x	longitudinal coordinate, m
Z	group of properties, Eq. (1)
$\tilde{Z}$	function of Z, Eq. (26)

Greek symbols

δ	spacing between branches, m
ΔP	pressure difference, Pa
ΔT	temperature difference, K
η	efficiency, Eq. (2)
η_p	pump isentropic efficiency
ν	kinematic viscosity, m^2 s^{-1}
ξ	dimensionless longitudinal coordinate
ρ	density, kg m^{-3}
ϕ	area fraction
φ	porosity
ϕ_c	critical area fraction

Subscripts

c	cold
f	fluid
p	plates
counter	counter flow
h	hot
in	inlet
min	minimum
opt	optimum
para	parallel flow

$$\eta_{max} = \frac{T_H - T_L}{T_H} \cdot \frac{Z - 1}{Z + T_L/T_H} \tag{1}$$

where Z (constant) represents a group of thermophysical properties of the material structure sandwiched between T_H and T_L. The energy conversion efficiency is defined as

$$\eta = \frac{I^2R}{q_H} \tag{2}$$

where I^2R is the delivered electrical power, and q_H is the total heat transfer rate from T_H to T_L. The power I^2R is associated with the flow of the generated current I through an external electrical resistance R.

The heat current q_H is proportional to the temperature difference that drives it,

$$q_H = \frac{T_H - T_L}{R_{th}} \tag{3}$$

where R_{th} is the thermal resistance of the TEM structure held between T_H and T_L. By combining Eqs. (1)–(3) we arrive at the important conclusion that maximum power is delivered in proportion with $(T_H - T_L)^2$,

$$(I^2R)_{max} = (T_H - T_L)^2 \cdot \frac{Z - 1}{R_{th}T_H(Z + T_L/T_H)} \tag{4}$$

The chief conclusion that follows from thermodynamics [1] is that maximum power is achieved in proportion with the temperature difference *squared*. When the volume of the TEM is fixed, maximum power also means maximum power *density*. The route to better designs with greater power density consists of maintaining the largest temperature difference across the TEM, between any point on the hot plate and its projection on the cold plate.

When the hot and cold plates are not isothermal, the design rule identified above becomes critical, and it applies at every point in the plane of the hot plate or the cold plate.

The largest temperature difference available is the difference between the inlet of the hot stream ($T_{h,in}$) and the inlet of the cold stream ($T_{c,in}$). Ideally, the thermo-fluid design of the TEM should be such that the hot plate is isothermal at $T_{h,in}$, and the cold plate is isothermal at $T_{c,in}$. This design would require thick high-conductivity bodies for the hot and cold plates, which increase the weight and decrease the power density of the TEM.

The challenge is to design the TEM when its hot and cold plates are large and not isothermal. This can be achieved by vascularizing the two plates such that, although nonuniform, the *local* temperature difference $(T_h - T_c)$ distribution is such that the integral of $(T_h - T_c)^2$ over the plane of the plate is the largest (Fig. 3).

2. Parallel flow, or counterflow?

Consider first the unidirectional flow configuration shown in Fig. 4. The hot stream flows as a sheet in one direction (x) parallel to the sheet flow of cold fluid. The capacity rates of the two streams are the same, $\dot{m}c_P$. The stream temperatures vary in the longitudinal direction, $T_h(x)$ and $T_c(x)$. When the streams are configured in parallel flow (Fig. 4), the first law of thermodynamics for each stream control volume of length dx requires