International Journal of Heat and Mass Transfer 79 (2014) 450–467

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Reliability-based design optimization of pin-fin heat sinks using a cell evolution method

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ARTICLE INFO

Article history:
Received 16 April 2014
Received in revised form 9 August 2014
Accepted 9 August 2014
Available online 2 September 2014

Keywords:
Pin-fin heat sink
Reliability-based design optimization
Cell evolution method
Thermal analysis
Finite element analysis
Uncertainties

ABSTRACT

This paper conveys the concept and idea of reliability-based design optimization (RBDO) to design a robust and reliable pin-fin heat sink for operating in an uncertain environment. In doing so, we formulate an RBDO heat sink design problem and incorporate a recently developed cell evolution method (CEM) to solve the formulated RBDO problem that aims to minimize entropy generation rate under the simultaneous variations caused by ambient temperature and the velocity of the air approaching the heat sink. From finite element analysis of the RBDO designed heat sinks, we found that the increment of the target reliability tends to create a more symmetric and uniform temperature distribution around the center of the heat sink and thereby presents a greater ability to resist the environmental variations. Besides, extensive simulation results reveal that the RBDO designed heat sink is superior to conventional deterministic ones in providing a much more robust and reliable heat dissipation performance when faced with environmental uncertainties. Furthermore, we report the visible findings about how the reliable solution varies depending upon the reliability indices for pin-fin heat sinks with inline and staggered arrangements.

1. Introduction

In recent years, the advanced semiconductor techniques have given rise to a size-reduced, high-tech device with an ever-greater system performance. This tendency of development inevitably leads to an increased heat generation rate per volume of the device. Consequently, a serious situation arises that the generated heat, if not appropriately removed, would greatly affect the normal operational performance of a working device; even worse, the device lifetime can be substantially shortened or might be severely damaged. Although diversified methods are available to avoid the overheating situation, up to date the use of air-cooling fins remains to be the most simplest and effective way to dissipate the generated heat of a high-tech device [1].

In the recent past, to achieve a better heat dissipation, there has been an increasing interest in proposing diversified optimization techniques to design air-cooling fins [2–30]. Bejan [2,3] introduced the entropy generation rate as the objective function in order to simultaneously maintain the minimal flow resistance and the maximal heat convection of a heat sink. By applying the method of entropy generation minimization (EGM), Iyengar and Bar-Cohen [4,5] considered the operating and production costs as an additional objective to optimize the performance of a plate-fin heat sink under the conditions of natural and forced heat convections. Based on minimizing a total entropy generation rate, Khan et al. [6–8] applied the method of Lagrange Multiplier to determine the optimal fin shape parameters as well as the operating conditions for pin-fin heat sinks. Culham and Muzychka [9] established a heat sink model by which an apparent friction factor is used to optimize a plate-fin heat sink equipped with a flow-through air inlet system. By performing constrained optimization on a CPU plate-fin heat sink, Shin and Liu [10] confirmed the superior performance of the optimized heat sink over some conventional designed ones which do not take the geometric constraints into the design consideration. In an optimization study of heat sink/fan assembly system, Mansuria and Kamath [11] concluded the necessity of implementing an air-cooling fan in the process to achieve a better performance of heat dissipation. Loh et al. [12] explored the optimal height of plate fins for operating in a deterministic, portable electronics operating environment. Krueger and Bar-Cohen [13] performed computer-aided designs to simultaneously determine the optimal shape, size, and pressure drop of a heat sink under forced convection. By minimizing the thermal resistance between the base device and the forced air streams, Bejan and Morega [14] determined the set of optimal geometries for a pin-fin heat

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.08.029
0017-9310/

Nomenclature

A	surface area of the base, m^2
A_b	exposed area of the base, m^2
A_c	the contact area of the fins, m^2
A_{fin}	surface area of each fin, m^2
C_p	specific heat capacity, J/kg K
C_1	a correction constant given by Eq. (12) or Eq. (18)
D	diameter of the fin, m
$\mathbf{d}$	the vector of the deterministic design variables
$\tilde{f}$	friction factor given by Eq. (15) or Eq. (19)
$f(\cdot)$	the objective function
$G_i(\cdot)$	the ith stochastic constraint
$g_j(\cdot)$	the jth deterministic constraint
H	height of the fins, m
h	heat transfer coefficient, W/m^2 K
h_b	the convective heat transfer coefficient given in Eq. (11), W/m^2 K
h_c	the effective heat transfer coefficient given in Eq. (5), W/m^2 K
h_{fin}	the convective heat transfer coefficient given in Eq. (10), W/m^2 K
K_1	a correction factor given by Eq. (16) or Eq. (20)
k	thermal conductivity, W/m K
k_f	the fluid thermal conductivity, W/m K
$k_{\max}$	the maximum number of generations
N	total number of pin-fins
N_{cell}	the number of reliability test cells
N_L	the number of rows in streamwise direction
N_T	the number of rows in spanwise direction
Nu_D	Nusselt number, $Nu_D \equiv h_{fin}D/k_f$
Nu_L	Nusselt number, $Nu_L \equiv h_b L/k_f$
$\dot{m}$	air flow rate, kg/s, defined by Eq. (13)
P_i	failure probability
Pr	probability function
$\tilde{\mathrm{Pr}}$	Prandtl number
$\mathbf{p}$	random parameters
p_c	crossover possibility
p_m	threshold of mutation
p_r	reproduction parameter
ΔP	the pressure drop across the fin, N/m^2, defined by Eq. (14)
Q	volume heat source, W/m^3
$\dot{Q}$	heat load, W
q	heat flux, W/m^2
R	reliability
R_b	thermal resistance of the base, K/W
R_c	the contact thermal resistance between fins and base, K/W
Re_D	Reynolds number, $\mathrm{Re}_D \equiv U_{app}D/\upsilon$
Re_L	Reynolds number, $\mathrm{Re}_L \equiv U_{app}L/\upsilon$
R_{fin}	thermal resistance of each fin, K/W
R_{fins}	the overall thermal resistance of fins, K/W
R_i	the reliability of satisfying the ith probabilistic constraint
R_m	the thermal resistance of the bulk material, K/W
R_{sink}	the overall thermal resistance of the heat sink, K/W
$\mathcal{S}_D$	dimensionless diagonal pitch, $S_D \equiv \mathcal{S}_D/D = \sqrt{S_L^2 + (S_T/2)^2}$
$\mathcal{S}_L$	dimensionless streamwise length, $S_L \equiv \mathcal{S}_L/D$
$\mathcal{S}_T$	dimensionless spanwise length, $S_T \equiv \mathcal{S}_T/D$
$\dot{S}_{gen}$	entropy generation rate, W/K
T	temperature, °C
T_{amb}	ambient temperature, K
t	time, sec
t_b	base width, m
U_{app}	the velocity of the air approaching the heat sink, m/s
U_{max}	maximum average velocity given by Eq. (17)
$\mathbf{x}$	the vector of the random (uncertain) decision variables
Z	sampling number for cell generation

Subscripts

amb	ambient
app	approach
b	base plate
c	contact
cell	cell
f	fluid
fin	single fin
fins	all fins with exposed base plate area
m	bulk material
sink	heat sink

Greek symbols

β	reliability index
ρ	density, kg/m^3
$\boldsymbol{\mu}_{\mathbf{x}}$	mean vector of $\mathbf{x}$
$\boldsymbol{\mu}_{\mathbf{P}}$	mean vector of $\mathbf{p}$
η_{fin}	efficiency of the pin-fin
υ	Kinematic viscosity, m^2/s
Φ	the standard normal cumulative distribution function
Ψ	joint probability density function
σ	standard deviation

sink to enhance the heat dissipation performance. In our previous studies [15,16], we analyzed the heat dissipation characteristics of heat sinks and applied a real-coded genetic algorithm to search for the optimal shapes and the set of operating conditions for plate- and pin-fin heat sinks equipped with flow-through and impingement-flow air cooling systems. Alternatively, Jubran et al. [17] explored the optimal inner fin spacing in both the spanwise and the streamwise directions through a series of experimental studies. By investigating the effects of pin-fin density on heat removal, Azar and Mandrone [18] ascertained an optimal fin number to maximize the heat dispersion performance of a heat sink process. Stanescu et al. [19] further explored the optimal cylinder-to-cylinder fin spacing by maximizing the overall thermal conductance. In addition, by investigating the effects of the geometrical configuration on heat dissipation, Tahat et al. [20] discovered an optimal distance between pin fins that is able to achieve a maximum heat transfer rate. Kondo et al. [21] presented a semi-empirical approach to search for an optimal pin-fin heat sink for impingement cooling of electronic packages. By extending the least-material analysis of single-fin to multiple-fin arrays, Bar-Cohen et al. [22–24] performed an optimization task to minimize the material usage of cylindrical pin-fins. They further carried out the least-energy optimization of air-cooled pin-fin heat sinks for geometry and material selection, as well as for the task of sustainable thermal management. Khan et al. [25,26] applied the EGM method to determine the optimal shape parameters and the flow condition of cylindrical pin-fin heat sinks. Recently, Ndao et al. [27] applied a multi-objective optimization approach to optimize the performance of a parallel micro-channel heat sink. In their design problem, the two objectives considered are the minimal total thermal resistance and the least pumping power consumption under the condition of a constant pressure drop. Alternatively, Shao et al. [28]