International Journal of Heat and Mass Transfer 79 (2014) 472–485

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Effects of density and thermal conductivity variations on entropy generation in gas micro-flows ☆

Sharad V. Prabhu [a], Shripad P. Mahulikar [a,b,*]

[a] Department of Aerospace Engineering, Indian Institute of Technology Bombay, P.O. IIT Powai, Mumbai 400076, India
[b] School of Engineering, Indian Institute of Technology, Mandi 175001, Himachal Province, India

ARTICLE INFO

Article history:
Received 17 January 2014
Received in revised form 26 June 2014
Accepted 20 July 2014
Available online 7 September 2014

Keywords:
Density variation
Entropy generation
Second law
Compressible flow
Thermal conductivity
Viscous dissipation

ABSTRACT

High temperature gradients in heat sinks of micro devices cause substantial properties variations in gas-micro-convection; which constitutes an important strategic research area in *non-rarefaction scaling effects*. In present analysis, compressible Navier–Stokes equations incorporating temperature dependent density and thermal conductivity variations, for steady, viscous flow of gas at subsonic speeds (Mach number $\ll 1$) are numerically solved. Circular micro-pipe geometry, subjected to constant wall heat flux was chosen. The key observations are: In addition to known effects, two additional physical mechanisms increasingly surface at micro scale; and also determine the micro convection characteristics within continuum regime. They are, induced radial convection due to density variations in radial and axial directions and axial conduction induced due to thermal conductivity variations along the flow. High density variations in radial and axial directions, cause velocity gradients and increase fluid friction irreversibility. High thermal conductivity variations cause flattening of temperature profile, induce axial conduction and substantially increase in entropy generation. The results highlight the need for incorporating fluid properties variation in thermal designs of heat sinks for micro devices. Two-way link between velocity profile and temperature, influence entropy generation. Corner effects at entrance and exit influence entropy generation in all cases studied.

1. Introduction

Gas micro convection is a strategic research area in transport phenomena, since it forms the basis of thermal designs of wide range of miniaturized, high performance applications like MEMS, micro-heat exchangers, micro reactors for chemical synthesis, laser diode [1], phased array radar [2], electronic payloads in spacecraft etc. Most of these devices are subjected to high temperature gradients and operate either in continuum, slip flow, or early transition regime with kn approximately < 0.1.

The deviations from the expected macro flow behavior are attributed to various scaling effects illustrated in Fig. 1. The physical mechanism for the scaling effects in gas-micro-convection [3] may be classified into: (a) *rarefaction effects*, (b) *non-rarefaction effects*.

As compared to rarefaction effects [4], relatively few studies dealing with non-rarefaction effects could be found in open literature. Therefore, the studies of non-rarefaction effects provide good research opportunities. The present study is focused on the influence of one of the important non-rarefaction effects, broadly classified as effects of variations in fluid properties. Such studies are computationally inexpensive and may help us in extending the limits of applicability of continuum assumption to higher Knudsen number regimes.

Some of the non-rarefaction scaling effects in gas micro convection include the effects of compressibility [5], viscous heating [6,7], axial conduction [8,9], surface roughness [10] and fluid properties variations [11].

Non-rarefaction effects increasingly surface towards micro scale due to changes in relative importance of the phenomena towards micro scale; and therefore, do not appear or are insignificant, in the conventionally sized channels. As an example, let us consider the axial conduction in a uniformly heated pipe. For this case, Peclet Number is defined as: $Pe_D = \rho_m C_p u_m D_h / k$; the inverse of Pe_D represents relative importance of axial conduction in a convective heat transfer flow. In the conventional pipe analysis, the axial conduction is usually negligible because ($Pe_D \gg 1$) (because

---

☆ The work has been carried out at I.I.T. Bombay.

* Corresponding author at: Department of Aerospace Engineering, Indian Institute of Technology Bombay, P.O. IIT Powai, Mumbai 400076, India. Tel.: +91 (0)22 25767122; fax: +91 (0)22 25722602.

E-mail address: spm@aero.iitb.ac.in (S.P. Mahulikar).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.07.062
0017-9310/© 2014 Elsevier Ltd. All rights reserved.

Nomenclature

Symbol

Be	Bejan number[–]
$Br_{q''_w}, Br_{sk}, Br_{s,\rho T}$	Brinkman number based on wall heat flux [–]. Br_{sk} and $Br_{s,\rho T}$ are modified Brinkman numbers based on temperature sensitivities of k and ρ resp.
C_p	specific heat at constant pressure [J/(kg K)]
D, *Dh*	diameter and hydraulic diameter of micro-pipe resp. [m]
Δe_y	percent deviation (error) for a given case from 'RhoK' case.
k	thermal conductivity [W/(m K)]
L	length of micro-channel [m]
$\dot{m}$	mass flow rate in a pipe [kg/s]
M	Mach number [–]
p	pressure at any point in the flow [Pa = N/m^2]; p_{exit} = Pressure at exit
Pe_D	Peclet number based on pipe diameter *D* [–].
Po	Poiseuille number [–]
Pr	Prandtl number [–]
q''_w	wall heat flux (constant) [W/m^2]
R	radius of micro-channel [m];
R_g	gas constant. (8.31451E3) [kg m^2/sec^2 kg mol K]
Re_D	Reynolds number based on diameter [–] at any point in flow.
$\dot{S}'''_{xxx}$	rates of local entropy generated per unit volume [W/(K m^3)] $\dot{S}'''_{HTrad}$, $\dot{S}'''_{HTaxi}$, $\dot{S}'''_{Vis}$, $\dot{S}'''_{Tot}$ due to heat conduction in radial, axial directions, fluid friction and total entropy respectively.
T, T_w	*T* is temperature at any point [K]; T_w is wall temperature [K].
T_m	bulk mean temperature at a cross section [K]
(*u*, *v*, *w*) or (V_z, V_r, V_ψ)	components of velocity vector $\overline{V}$ in coordinate directions [m/s]. Sometimes, also denoted by (V_z, V_r, V_ψ).

Greek symbols

ρ	density [kg/m^3]
μ	dynamic viscosity [Pa s]
v	kinematic viscosity = (μ/ρ) [m^2 s]
τ	wall shear stress [Pa]
$\varnothing$	viscous dissipation function [W/m^3]
Φ	Irreversibility distribution ratio [-].
r, ψ, *z*	cylindrical coordinate system for the present problem.

Superscripts†

· (dot), ′ and ‴ and *	superscript dot (·) indicates time rate. Single ′ indicates quantity per unit length; ‴ indicates quantity per unit volume, * indicates dimensionless quantity
bar	bar over variable indicates dimensionless variable e.g. ($\bar{u} = u/u_m$)

Subscripts

m	mean quantity over cross section. e.g. u_m
w, 0	the parameter value at wall and axis of micro-tube resp.

of large *D*!); while it becomes non-negligible in gas-micro-convection and cannot be ignored (Mahulikar [8]).

One important non-rarefaction effect, that warrants special attention in the context of thermal designs of micro devices, is the influence of thermo-physical properties variations on the flow.

To prove the point, let us consider a micro flow device operating in the temperature range from 275 K to say 600 K. The data for dry air heated case [14] suggests that variations in temperature dependent fluid properties in this range are estimated to be of following order:

$k(T)$ = 75%, $\rho(T)$ = 50%, $\mu(T)$ = 75% and $C_p(T)$ < 5%. These results suggest that, the fluid properties variations in the flow are substantial. In present exercise it will be subsequently proved that their influence on gas-micro-convection is significant.

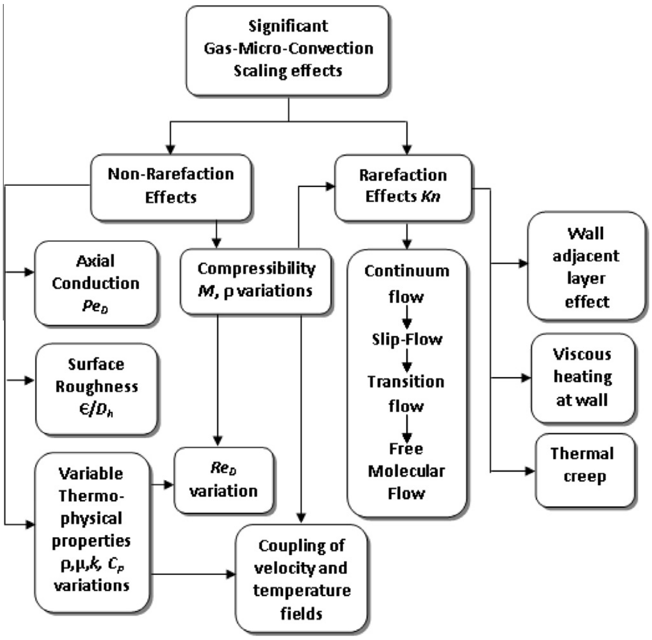

Fig. 1. Rarefaction and non-rarefaction effects in gas micro convection.

From the available published literature related to the influence of properties variation in gas micro convection, some typical, representative studies are discussed here briefly:

Mahulikar et al. [15] have shown that steep density gradients cause flattening of axial velocity profile which is similar to constant property *slip* flow and cause flow *un*-development. Lee et al. [16] numerically studied the effects of variable thermal conductivity on thermal enhancement of water flow in silicon micro channel. Their study concluded that the temperature dependent variable fluid properties influence the velocity and temperature fields significantly. Guidice et al. [17] numerically studied the effects of viscous dissipation and temperature dependent viscosity in thermally developing laminar flow of liquids in straight micro channels in terms of the axial distributions of local Nusselt number and Fanning friction factor, for both conditions, heating and cooling. Al-Zaharnah et al. [18] studied the influence of variable viscosity on entropy generation for turbulent water flow through steel pipe. Gozukara et al. [19] numerically analyzed the combined effects of (μ, k) variations in developing, single phase, laminar, incompressible air flow in a micro gap between parallel plates. Sun and Jaluria [20] studied the effect of pressure work and viscous dissipation on convective heat transfer characteristics in pressure-driven nitrogen slip flows in long micro channels; and concluded that pressure work and viscous dissipation significantly influence the Nusselt Number in continuum as well as rarefaction regimes. Recently Mahulikar et al. [11] numerically studied the pressure drop characteristics in laminar convective flow covering variations