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New delay differential equation models for heating systems with pipes

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A R T I C L E I N F O

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A B S T R A C T

There is no doubt on the importance to investigate and develop the efficiency of engineering systems utilizing thermal energy. For this purpose, mathematical modeling is the theoretically established tool to analyze such systems. In this study, new, validated delay differential equation (DDE) models are proposed for rather general heating systems with a heater, pipes and a storage (and optionally a heat exchanger). The flow rate in the pipe, and thus the time delay, can be either time dependent or state dependent. The advantages of the proposed DDE models, compared to the conventional ordinary differential equation (ODE) and partial differential equation (PDE) approaches, are discussed. Based on the comparison of measured and simulated data of a real solar heating system, the validation and the efficiency of the proposed DDE models are demonstrated.

Applicability of the models is also discussed along with future research proposals.

1. Introduction

Nowadays, there is no doubt on the importance to investigate and develop engineering systems utilizing thermal energy (from some renewable or non-renewable resource). Mathematical modeling is the most widely used and theoretically established tool for this purpose.

In this study, thermal engineering systems are investigated having a heater and a storage (and optionally a heat exchanger) as the main working components. The neighboring components are connected with pipes, through which a pump circulates some heat transfer fluid (see Figs. 1 and 4). Thermal engineering systems corresponding to this basic type, and called simply *heating systems* in this study, cover a wide variety of thermal energy installations such as central heating systems, district heating systems, solar heating systems etc.

In the literature, the mathematical models of heating systems are often linked with specific systems (e.g. solar heating systems), nevertheless, it is usually easy to extend the corresponding results to more general systems.

In many works, the delaying effect of the pipes is neglected. This is the case, when the heating systems are modeled with conventionally used ODEs. In [1,2], heater-storage systems without a heat exchanger are modeled in such a way. The heater is a solar collector in [1]. In [2], the heater and the storage are divided into several layers, which are characterized with homogeneous temperatures forming a multidimensional ODE. Natural circulation (with no pumps) is considered in the work. In [3,4], heater-heat exchanger-storage systems are modeled with a linear ODE, the nonlinear version of which can be found in [5].

In [6], 5–10% more solar energy is utilized in a real measured solar heating system with the same control type (differential control) when the effect of the pipes is considered, so it may be very important to model this effect. The delaying (and optionally the heat loss) effect of the pipes can be taken into account with pipe discretization. The TRNSYS software [7] is widely used for the simulation of transient thermal processes in heating systems. In this tool, the pipes are divided into segments with different homogeneous temperatures, each of which is modeled with an ODE (providing a cell approximation of a corresponding PDE model). The following are examples for heating system simulations with the TRNSYS: [6,8,9]. In [10], a partial differential equation (PDE) that is the linear transport equation corresponding to mass density is used in modeling specific total energy of the fluid in district heating systems. A further PDE resulted from this approach is solved along the characteristic path determined by the fluid particle motion. Then the pipes are discretized, the temperature of which is calculated numerically. Another discrete approach can be found in [11], where an air conditioning system is considered as a dynamical system evolving in discrete time steps. The used discrete model is the so-called cellular automaton. The pipes of the system are composed of finite cells with equal fluid mass, based on which the outlet temperature is modeled.

The most frequently used model, with which the delaying (and heat loss) effect of a pipe is considered, is the one dimensional

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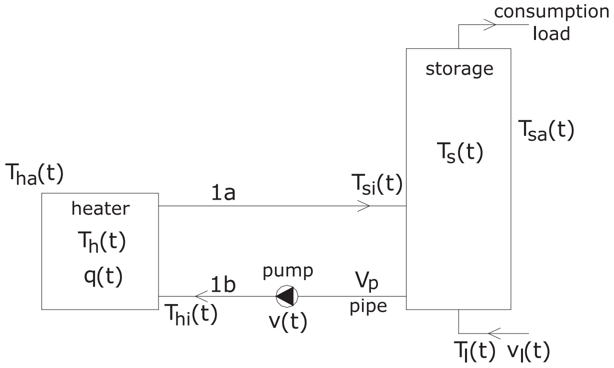

Fig. 1. Scheme of the heating system without a heat exchanger.

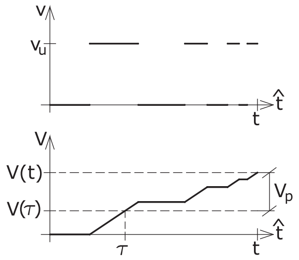

Fig. 2. Determination of $\tau(t)$.

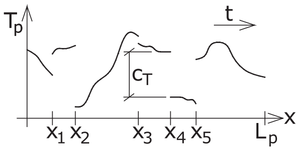

Fig. 3. Discontinuities in the pipe temperature along the pipe.

linear heat transfer PDE corresponding to plug-flow. Namely, the model is the one dimensional linear transport equation corresponding to the pipe (fluid) temperature (see Eq. (8) below), completed with a member for heat loss if needed. This PDE pipe model is used to describe pipe temperature distribution inside the collector of a simple solar heating system in [12], inside single- and double-side heat exchangers along with connecting pipes under negligible heat losses in [13], inside an industrial chemical tubular continuous reactor in [14], inside the control volumes, into which a built-in-storage solar water heater is divided in [15], inside a flat-plate solar collector in [16] and inside a district heating system in [17]. In [17], the PDE model is also used to describe time delay. In [18], the pipe wall temperature distribution is also described with a further PDE associated with the already mentioned PDE corresponding to the pipe fluid. In [19], the PDE for pipe fluid temperature distribution is applied for a pilot-scale water heating equipment with perfectly insulated pipes. The PDE is transformed into two simplified model for control purposes. One of the models consists of ODEs for perfectly mixed sections and the other is a length integrated model, which determines the average (fluid) temperature along the pipe. Also, PDEs corresponding to energy balance are used to describe temperature distribution inside parallel-plate channels and pipes in [20] and inside a basic natural circulation loop in [21].

Few examples can be found in literature working with delay equation models containing directly the time delay of a pipe for describing the temperature distribution or the outlet temperature as a function of time [22], although the delay approach is often more natural and convenient in handling such problems than the more conventional PDE and ODE approaches (see Remarks 2.1 and 4.2 below). In [22,23], the heat transfer PDE is used to derive a delay equation, which describes the pipe outlet temperature in time in case of constant and variable flow rates, respectively. These works contain relatively simple delay equations for a single pipe.

The application of delay differential equations (DDEs) is the natural extension of the delay approach for complex heating systems. (See e.g. [24,25] for the general theory of DDEs.) Most works on thermal engineering problems belong to the case of constant time delay. See e.g. [26] on the design of a control to regulate the heating of living tissue based on a nonlinear DDE, [27] and [28] on the DDE model of heat conduction in case of that there is a time delay of the heat flux vector and [29] on the DDE model of mixed problems for diffusion and reaction–diffusion equations. See [30] on a DDE model (of the so-called neutral type) for combustion systems. Some of the above works also serve with PDEs with delays.

After the mentioned physically-based models, it is finally worth mentioning that heating systems can be also described with heuristic models, which consider the whole system as a black-box (without dividing it into a heater, pipes, etc.) and try to recognize rules on the output(s) as a function of the input(s). Such approaches can be found e.g. in [31] and [32], where artificial neural networks (collection of individually interconnected processing units) are used and in [32], where genetic algorithms (an optimum search technique) are applied to model heating systems.

In this study, new DDEs for rather general heating systems with or without a heat exchanger are considered. The pump flow rate (in the pipes), and thus the time delay can be time dependent or state

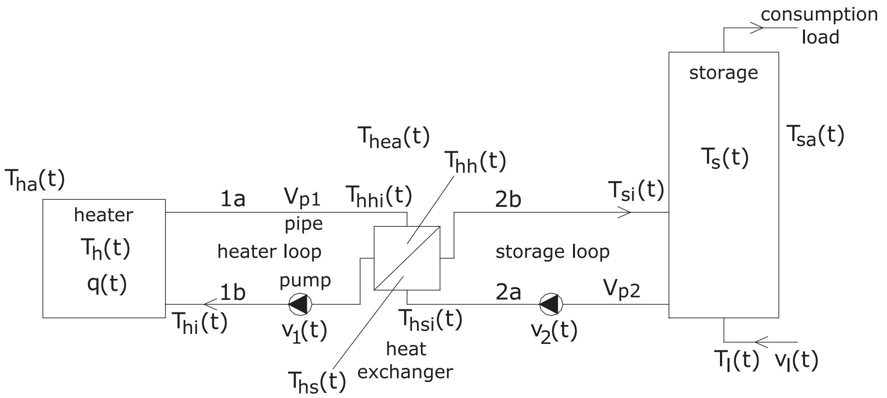

Fig. 4. Scheme of the heating system with a heat exchanger.