International Journal of Heat and Mass Transfer 79 (2014) 829–837

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Characteristics of an evaporating thin film of a highly wetting liquid in a groove

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ARTICLE INFO

Article history:
Received 11 April 2014
Received in revised form 13 June 2014
Accepted 3 August 2014
Available online 18 September 2014

Keywords:
Evaporation
Thin film
Highly wetting liquid
Marangoni effect
Groove
Numerical analysis

ABSTRACT

The behavior of an evaporating thin film of a highly wetting liquid filled in a rectangular groove was studied theoretically. Following the previous studies, the liquid film was divided into an adsorbed film region, a micro region and an intrinsic meniscus region. A theoretically based method was employed to estimate the Hamaker constant. The Marangoni effect due to the thickness change of a very thin liquid film was included in the basic equation. The fourth order differential equation for the liquid film thickness was solved by the finite difference method. Three sets of the combination of boundary conditions at both ends of the micro region were examined and the set of boundary conditions which gave the most realistic solution was decided. The solutions were obtained for a wide range of the micro region length. The principle of maximum entropy production was introduced to estimate the most probable solution. The results were compared with previous results obtained by the shooting method and the effect of the solution method on the numerical results was discussed.

1. Introduction

Heat transfer during evaporation from a very thin liquid film plays a very important role in thermal equipments such as grooved heat pipes, microchannel evaporators and microfin tube evaporators, and in nucleate boiling. The behavior of thin liquid films is considerably different from that of thick films due to the effects of long-range intermolecular forces. A comprehensive review of relevant literature is given by Wayner [1]. Potash and Wayner [2] presented a theoretical model of evaporation from the extended meniscus of a highly wetting liquid on a vertical wall, which consisted of an adsorbed film region with no evaporation, a micro region with a very thin liquid film and an intrinsic meniscus region. A number of investigators extended this model to the analysis of heat transfer in the grooved heat pipes [3–7], microchannel evaporators [8] and microfin-tube evaporators [9]. This model was also used for the analysis of heat transfer during nucleate boiling [10–12] and evaporation of liquid droplets [13].

The basic equations for the evaporating thin liquid film are reduced to the fourth order ordinary differential equation for the liquid film thickness δ with respect to the coordinate x measured along the heated surface. At the boundary between the adsorbed film region and the micro region, $x = 0$, where the evaporation rate is zero, the boundary conditions are given by [7,8]

$$\delta = \delta_0 \quad (1)$$

$$d\delta/dx = 0 \quad (2)$$

$$d^2\delta/dx^2 = 0 \quad (3)$$

$$d^3\delta/dx^3 = 0 \quad (4)$$

where δ_0 is the thickness of the adsorbed film. According to Wayner [1], δ_0 is given by

$$\delta_0 = \left(\frac{-AT_v}{6\pi\rho_l h_{lv}\Delta T}\right)^{1/3} \quad \text{for } \delta_0 < 10\,\text{nm} \quad (5)$$

where A is the Hamaker constant of the system consisting of vapor, liquid film and solid substrate. Solution of the above mentioned fourth order differential equation subject to the boundary conditions given by Eqs. (1)–(4) results in the trivial solution of $\delta = \delta_0$ all over the surface [7,8]. In order to avoid this difficulty, Xu and Carey [3] and Ha and Peterson [6] adopted a simplified model in which the capillary effect on the liquid pressure was neglected. Khrustalev and Faghri [5] adopted a simplified model in which the radius of curvature of the evaporating liquid film r was assumed to be equal to that of the intrinsic meniscus r_b all over the micro

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.08.044
0017-9310/

Nomenclature

A	Hamaker constant of vapor–liquid film-substrate system, J
B_1, B_2	parameters in Eq. (29)
h_{lv}	latent heat of evaporation, J/kg
m	mass flow rate of liquid, kg/m
P	pressure, Pa
P_d	disjoining pressure, Pa
P_m	recoil pressure, Pa
ΔP	total pressure difference ($= P_v - P_l$), Pa
Q	heat flow rate, W/m
q	heat flux, W/m^2
r	radius of curvature of liquid film, m
T	temperature, K
ΔT	wall superheat ($= T_w - T_v$), K
w	groove width, m
x, y	coordinates along and normal to the groove wall, respectively, Fig. 1, m

Greek symbols

δ	liquid film thickness, m
ε	angle, Fig. 1, °
λ	thermal conductivity, W/m K
μ	dynamic viscosity, Pa s
ρ	density, kg/m^3
σ	surface free energy, J/m^2

Subscripts

b	connecting point between micro region and intrinsic meniscus region
c	bottom of intrinsic meniscus
i	vapor–liquid interface
l	liquid
max	maximum value
new	new value
old	old value
v	vapor
w	wall
0	adsorbed film

region. Stephan and Busse [4], Wang et al. [8] and Stephan and Hammer [10] solved the set of basic equations by the shooting method, in which one of the boundary values at $x = 0$ was perturbed so that r approached r_b at a large x. Do et al. [7] solved the basic equations with the boundary conditions of

$$r = r_b \tag{6}$$

$$d\delta/dx' = -\tan\varepsilon \tag{7}$$

at $x' = 0$, where $-\tan\varepsilon$ was the slope of the liquid film at the boundary between the micro region and the intrinsic meniscus region and x' was the coordinate measured in the $-x$ direction from the boundary. The value of r_b was assumed to be a function of the liquid film thickness at $x' = 0$. The end of the micro region was determined by the condition that the evaporation rate there was equal to 0. Among the solution methods described above, that of Stephan and Hammer [10] is considered to be most accurate, because it does not include additional assumptions for the boundary values except for the choice of the perturbed parameter. Honda and Wang [9] solved the fourth order differential equation for the liquid film thickness by the finite difference method. The boundary conditions at $x = 0$ were Eq. (2) and

$$\delta = \delta_{0r} \tag{8}$$

where δ_{0r} ($> \delta_0$) was the thickness of the liquid film considering the curvature effect. The geometrical condition that the liquid film in the micro region was smoothly connected with the intrinsic meniscus region at $x = x_b$ was given by Eq. (6) and

$$d\delta/dx = \tan\varepsilon \tag{9}$$

This solution method is attractive because the boundary conditions at both ends of the micro region are satisfied correctly. However, the obtained solution was not realistic on the point that, at $x = 0$, the liquid film thickness was not continuous and the curvature of the liquid film took the maximum value. Thus more work is required to decide the combination of boundary conditions at $x = 0$ and $x = x_b$ which gives the most realistic solution.

As described in [1], the value of the Hamaker constant depends on the combination of vapor, liquid and solid substrate which constitutes the evaporation system. In the previous studies, however, sufficient information on the physical properties required to calculate A was not available. In the present study, the value of A is estimated using a theoretically based method. Another problem that has not been addressed in the previous studies is the Marangoni effect which was caused by the surface energy change of a very thin liquid film along the surface as a result of the change in the liquid film thickness. In the present study, the interfacial shear stress due to the Marangoni effect is taken into account in the basic equation. The fourth order differential equation for the liquid film thickness is solved by the finite difference method developed by Honda and Wang [9]. Numerical calculations are conducted for three sets of the combination of boundary conditions at $x = 0$ and $x = x_b$ and the numerical results are compared with each other. Numerical solutions are obtained for the evaporation of FC-72 filled in a small rectangular groove made of copper. Comparison is made between the cases in which the Marangoni effect is taken into account and neglected. Comparison is also made with the previous results of Raj et al. [13] for the evaporation of a FC-72 droplet on a heated surface in which the shooting method of Stephan and Hammer [10] was adopted.

2. Theoretical model

Consider evaporation of a highly wetting apolar liquid filled in a rectangular groove. Fig. 1 shows the physical model and the coordinates. The coordinate x is measured downward along the groove from the boundary between the adsorbed film region and the micro region, and the coordinate y is measured vertically outward from the groove surface. Depending on the amount of liquid filled in the groove, the coordinate $x = 0$ is located at the edge of the groove or on the groove wall. The coordinate x_c denotes the position of liquid meniscus at the center of the groove. For the micro region, the momentum equation in the x-direction is written as

$$\mu_l \frac{d^2u}{dy^2} = \frac{dP_l}{dx} \tag{10}$$

The boundary conditions are

$$u = 0 \quad \text{at } y = 0 \tag{11}$$