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Instability of three-dimensional flow due to rotation and surface-tension driven effects in a shallow pool with partly free surface

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ABSTRACT

The fundamental characteristics of the three-dimensional flow induced by rotation and surface-tension driven effects subjected to a horizontal temperature gradient in a shallow cylindrical pool with a disk on the free surface are investigated through a series of numerical simulations. The aspect ratio (height/radius) of the system considered in this work is 0.06 and the radius ratio is 0.3. The results indicate that the basic flow is axisymmetric and steady. It behaves as rich flow structures in the meridian plane. However, with the increase of the rotation and thermocapillary Reynolds numbers, the flow will undergo a transition to a three-dimensional oscillatory flow, which is characterized by the temperature and velocity fluctuation waves traveling in azimuthal direction. The direction and velocity of wave propagation, fluctuation amplitude and wave number are dependent on the interactions of the thermocapillary, centrifugal and Coriolis forces. The critical conditions for the onset of flow instabilities are obtained. The stability diagram is presented, which shows the critical thermocapillary Reynolds number varies with the rotation rates of the pool and disk. In particular, when the disk counter-rotates with the pool, three different flow states are observed and mapped with different thermocapillary Reynolds numbers. Besides, the origins of these flow instabilities in different unstable state regimes are discussed and briefly summarized in this paper.

1. Introduction

The complex flow driven by rotation and temperature gradient has been the subject of many studies, not only a fundamental interest as prototype flow for three-dimensional (3-D) rotating flow but also a topic of practical importance in the performance improvement of many industrial devices. In industry, an important example is the Czochralski (Cz) crystal growth technology where both the crucible containing the melt and the crystal growing at the melt surface are rotated. Thus, the forces that can drive the flow include the thermocapillary, buoyancy, centrifugal and Coriolis forces. These forces interact on different scales making the Cz crystal growth process difficult to control and characterize. Several flow instabilities may occur and have a direct impact on the quality of the growing single crystal, such as the undesired creation of striations. Therefore, the instability behaviors during Cz crystal growth process have been extensively studied and the literature that referred to here is by no means exhaustive.

The thermocapillary flow instability was first predicted by Smith and Davis [1]. They performed a linear stability analysis of a thin and infinitely extended fluid layer with free upper surface subjected to a horizontal temperature gradient, and found two types of thermal convective instabilities depending on the Prandtl (*Pr*) number and the basic flow pattern. Later, many researchers devoted to study the thermocapillary and thermocapillary–buoyancy convections [2–4]. And it was indicated that the thermocapillary flow exhibits several types of 3-D instabilities. Subsequently, Shi et al. [5,6] performed linear stability analysis and nonlinear numerical simulations about the effect of pool rotation on the basic thermocapillary flow and the critical conditions for the onset of the hydrothermal waves in a rotating annular pool heated from the outer wall. The test fluids of silicone oil ($Pr = 6.7$) and silicon melt ($Pr = 0.011$) were both investigated. Results showed that the pool rotation destabilizes the basic steady axisymmetric thermocapillary flow. Recently, the research conducted by Gelfgat [7] reported the explanation of the destabilization effect of weak rotation.

On the other hand, experimental studies of instabilities of the melt flow in Cz configuration has been carried out either by measuring the temperature or velocity oscillation at some fixed points into the melt, or by visualization of the melt flow on the free

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Nomenclature

A	aspect ratio
f	frequency, Hz
F	dimensionless frequency
h	depth, m
H	dimensionless depth
m	wave number
n	rotation rate, rpm
P	dimensionless pressure
Pr	Prandtl number, $Pr = \nu\alpha$
r	radius, m
R	dimensionless radius
Re	Reynolds number
T	temperature, K
V	dimensionless velocity vector
z	axial coordinate, m
Z	dimensionless axial coordinate

Greek symbols

α	thermal diffusivity, $m^2\ s^{-1}$
β	growth rate constant
γ_T	temperature coefficient of surface tension, $N\ m^{-1}\ K^{-1}$
Θ	dimensionless temperature, $\Theta = (T - T_c)/(T_h - T_c)$
λ	thermal conductivity, $W\ m^{-1}\ K^{-1}$
μ	dynamic viscosity, $kg\ m^{-1}\ s^{-1}$
ν	kinematic viscosity, $m^2\ s^{-1}$
ρ	density, $kg\ m^{-3}$
τ	dimensionless time
ψ	dimensionless stream function

Subscripts

c	cylindrical pool or critical or cold
h	hot
s	disk
w	angular velocity
R, Z, θ	coordinate directions

surface or inside the container using transparent liquid and crucible for a model experiment [8,9]. Fein and Pfeffer [10] conducted an experiment to investigate the thermal convection in a rotating annulus and found the baroclinic waves in a rotating annulus have a drift direction counter to the annulus rotation in the low-rotation regime and a co-rotation in the high-rotation regime. Contrarily, Seidl et al. [11] observed that the azimuthal m-folded ($m = 2,3,4,\ldots$) waves propagate in a direction opposite to the crucible at higher crucible rotation rates by both experiments and numerical simulations. With the higher crucible rotation rate, the wave number is larger and the angular velocity of the wave is faster, but always lower than the crucible rotation rate [12]. Lee and Chun [13] also showed that the thermal waves travel in the azimuthal direction when the rotation rate of the crystal exceeds a certain critical value but disappear for sufficiently high value in a Cz configuration, considering the coupling of the buoyancy- and the crystal rotation-induced flows. Furthermore, Hintz and Schwabe [14] reported that the flow will transit to oscillatory convection once the crystal rotation rate exceeds the threshold value, which is a monotone increasing function of the temperature difference between the crystal and the crucible. For both rotation of the crystal and the crucible with a horizontal temperature gradient, Son et al. [15,16] found that the effect of counter-rotation of the crystal and crucible provides the velocity reduction of the oscillatory wave. As the crucible rotation rate increases, the maximum thermal fluctuation region migrates toward the crucible sidewall. However, when the crystal counter-rotation rate increases, the thermal fluctuation at the edge of the crystal is reduced.

Since it is difficult to observe the details of the convection by experiments, numerical simulations have been carried out to understand the characteristics of thermocapillary flow combined with crystal or/and crucible rotation [17]. Rojo and Derby [18] performed the high-resolution calculations using a parallel finite element method to simulate the flow of molten bismuth silicon oxide driven by the combined rotational and buoyant force during the Cz crystal growth process. For the first time, the spoke patterns were predicted along the surface of the melt at high crystal rotation rates. These patterns corresponding to radial aligned roll cells are confined to a thin layer near the melt surface, which aroused from a modified Rayleigh instability within a destabilizing thermal boundary layer caused by crystal rotation and centrifugal pumping. Subsequently, 3-D numerical simulations have been performed for hydrodynamic model of the Cz crystal growth [19]. The results showed that asymmetrical phenomena of the fluid flow and the critical temperature difference for the flow pattern transition are dependent on the crystal and crucible rotations. However, Gunzburger et al. [20] reported that the crucible and crystal rotations are ineffective in reducing the velocity perturbation, as well as the temperature gradient in the Cz configuration. With these divergences, Shi et al. [21] and Li et al. [22] reported the results of 3-D numerical simulations of the thermocapillary flow for silicone oil and silicon melt in a rotating shallow annular pool, respectively. It was certified that the rotation rate of the annular pool has an important effect on the critical temperature difference and the propagating direction of the traveling waves.

As mentioned above, in the Cz crystal growth process, the melt flow is strongly influenced by the thermocapillary force, buoyancy force, centrifugal and Coriolis forces induced by the rotation. However, even when the flow is driven by rotation only, the flow field represents as beautiful patterns and attracts the interest of physicists as examples of pattern formation in flow [23–26]. Kanda [27] performed an experiment on the instability behaviors in a differential rotation system derived from the Cz crystal growth without thermocapillary and buoyancy forces. He observed various flow patterns as the increasing rotation rate difference between the crystal and the crucible. To extract the effect of rotation only during the Cz process, Li and colleagues [28,29] numerically investigated the detailed characteristics of the forced flow driven by iso- and counter-rotation of crystal and crucible regardless of the temperature gradient. It is indicated that the rotation-driven flow will undergo a transition from the axisymmetric and steady state to the 3-D oscillatory flow when the rotation Reynolds number exceeds a critical value.

It is known that when the temperature gradient exceeds a threshold value, the pure thermocapillary flow in a thin annular pool [30] or in a shallow Cz configuration [31] becomes unstable and the fluctuations are traveling in the azimuthal direction with a constant angular velocity. The mechanism of flow transition is the hydrothermal wave instability which only occurs in a shallow system [1]. Also, it is proved that in the shallow Cz configuration, the buoyancy effect is negligible [31]. On the other hand, as reported in our previous paper [28], several instabilities were observed for