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Analysis of bifurcations in a Bénard–Marangoni problem: Gravitational effects

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ABSTRACT

This article studies the linear stability of a thermoconvective problem in an annular domain for different Bond (capillarity or buoyancy effects) and Biot (heat transfer) numbers for two set of Prandtl numbers (viscosity effects). The flow is heated from below, with a linear decreasing horizontal temperature profile from the inner to the outer wall. The top surface of the domain is open to the atmosphere and the two lateral walls are adiabatic. Different kind of competing solutions appear on localized zones of the Bond–Biot plane. The boundaries of these zones are made up of co-dimension two points. A co-dimension four point has been found for the first time. The main result found in this work is that in the range of low Prandtl number studied and in low-gravity conditions, capillarity forces control the instabilities of the flow, independently of the Prandtl number.

1. Introduction

It is well known that two different effects are responsible of the thermoconvective instabilities in fluid layers: gravity and capillarity forces. The problem in which both effects are considered, known as Bénard-Marangoni (BM) convection, has become a classical problem in fluid mechanics [1]. In the classical BM problem, heat is uniformly applied from the bottom wall and the solution becomes unstable for increasing temperature gradients. A more general problem includes the effect of non-zero horizontal temperature gradients arising new thermoconvective instabilities. These instabilities have been analyzed considering both a rectangular domain containing the flow [29,19,11,26,6,3,20,24], or an annular geometries [14,8,15,10,21,12].

In order to characterize the different effects steering the behavior of the flow, the following set of dimensionless numbers has been introduced:

1. Aspect ratio, $\Gamma = \delta/d$. Is the geometrical parameter that characterizes the domain.
2. Marangoni number, $Mar = \gamma \Delta T d^2/\rho\kappa\nu$: Characterizes the surface tension effects.
3. Prandtl number, $Pr = \nu/\kappa$: The ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. In this article several different Pr values will be considered covering the different situations concerning the momentum diffusivity to the thermal diffusivity.
4. Rayleigh number, $Ra = g\alpha\Delta T d^4/\kappa\nu$: Representative of the buoyancy effect.
5. Biot number, Bi: Accounts for heat transmission between the fluid and the atmosphere. Values inside the range [0.2–3.2] are explored in this article.
6. Bond number, $Bo = Ra/Mar = g\alpha\rho d^2/\gamma$: Ratio of Rayleigh to Marangoni numbers, which is the control parameter in this analysis ranging from $Bo = 0 \rightarrow g = 0.0$ to $Bo \approx 64 \rightarrow g = 9.9$.

In the previous definitions, δ and d are characteristic lengths of the domain that will be defined in the following section; γ stands for the rate of change of surface tension with temperature; ΔT is the temperature increment, ranging from 2 to 50; ρ, κ, α and ν are the density, the thermal diffusivity, thermal expansion coefficient and the kinematic viscosity of the fluid, respectively; and g is the acceleration due to gravity.

The importance of heat-related parameters on the development of instabilities was analyzed in [15,14]. More recently, the problem was also studied in an annular geometry [25,27,28] but neglecting the effect of Biot number and considering conduction through the lateral walls of the cylinder. Hoyas et al. [16] analyzed the effect of

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Nomenclature

List of symbols

a	diameter of the internal cylinder
d	diameter of the physical domain
e_z	unit vector in z direction
g	acceleration due to gravity
k	wave number
p	pressure
r	radial coordinate
T_n, T_m	Chebyshev polynomials
T_G	temperature gradient at the bottom surface
ΔT	mean temperature difference between the bottom plate and the atmosphere
$\mathbf{u}$	velocity vector
u_j	velocity components (r, ϕ, z)
z	z coordinate
α	thermal expansion coefficient
γ	rate of change of surface tension with temperature
δ	thickness of the physical domain
κ	thermal diffusivity
Λ	aspect ratio
λ	real part of the eigenvalue (stability)
ν	kinematic viscosity
ρ	density
Θ	remperature
ϕ	Azimutal coordinate

Dimensionless numbers

Bi	Biot number
Bo	Bond number, $Bo = Ra/Mar = g\alpha\rho d^2/\gamma$
Mar	Marangoni number, $Mar = \gamma\Delta T d^2/\rho\kappa\nu$
Pr	Prandtl number, $Pr = \nu/\kappa$
Ra	Rayleigh number, $Ra = g\alpha\Delta T d^4/\kappa\nu$

Sub- and superscripts

c	critical value

Biot number on the different bifurcations for the case of $Pr = \infty$. The computational method was validated by comparing the results obtained with the experimental results by Garnier et al. [10]. The computational method has been recently modified [30] to be used with Prandtl numbers close to unity. In [12], the authors investigated the existence of co-dimension three bifurcations that are the points where the co-dimensions two curves intersect on the Prandtl-Biot plane and an also new kind of instability was predicted. Those latter works dealt with the influence of Biot number in the flow solutions. The interest in understanding the influence of gravitational effects in thermo-convective phenomena has been rapidly growing [7,23]. However, less attention has been paid to the effect of the capillarity forces of the onset of the flow motion and the behavior of the bifurcations that can appear.

The current work is devoted to obtain a deeper insight on the effect of the gravitational and capillarity forces of the onset of the flow motion, keeping in mind that understanding this flow behavior will contribute open a gateway to control the instabilities. To achieve this goal, a linear stability analysis, similar to the one in [16], will be performed, but instead of focusing on the influence of the Prandtl number, the focus will be put on understanding the effect of variations in the gravitational forces (or what is the same for a fixed geometry, the Bond number). Simulations will be performed in two different ranges of Prandtl number: $Pr = 1$ and $Pr = 50$. In [12] it was shown that Pr is the main parameter to determine the shape of the growing solution. This also applies to the current problem, but in the case of low gravity conditions, the dependency on Prandtl of the Rayleigh and Marangoni numbers is far less clear.

The paper is structured as follows. In the second section the formulation of the problem is presented, and in the third one the numerical method used to solve it. Then, in the fourth section the results are discussed. In the fifth and last section conclusions are presented, and future works are proposed. As this work contains many adimensional numbers and parameters, a list of symbols has been added before the bibliography.

2. Model description and formulation

The physical domain considered in this work consists of a horizontal fluid layer of depth d (z coordinate) which is contained in the annular ring limited by two concentric cylinders of radii a and $a + \delta$ (r coordinate). A sketch of the domain is presented in Fig. 1. The aspect ratio, Γ, is set to 4 and the diameters of the two cylinders are chosen so that the bigger is the double of the smaller one ($a = \delta$). The bottom surface is considered to be rigid and is heated with a linear decreasing temperature gradient with a value of $T_G = 2.2$ K, which is kept constant throughout this study. The top surface is open to the atmosphere and the two lateral walls of the cylinder are considered adiabatic. The reference temperature used in the definition of the Rayleigh and Marangoni numbers is the mean temperature difference between the bottom plate and the atmosphere, ΔT.

The fluid layer behavior can be described by means of the momentum and mass balance equations and the energy conservation principle. These equations expressed in cylindrical coordinates and non-dimensionalized as in [15,5] become

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = Pr\left(\nabla p + \nabla^2 \mathbf{u} + Ra\Theta \mathbf{e}_z\right), \tag{2}$$

$$\partial_t \Theta + \mathbf{u} \cdot \nabla \Theta = \nabla^2 \Theta. \tag{3}$$

In the equations governing the system u_r, u_θ and u_z are the components of the velocity field $\mathbf{u}$, Θ is the temperature, and p is the pressure. In these equations the operators and fields are expressed in cylindrical coordinates and $\mathbf{e}_z$ is the unit vector in the z direction. The Boussinesq approximation has been used as it is usual in this sort of problem [5]. Boundary conditions are similar to those of Refs. [15,30] and are summarized in Table 1. Briefly, the velocity is zero (no-slip wall condition) on the lateral walls and the bottom plate.

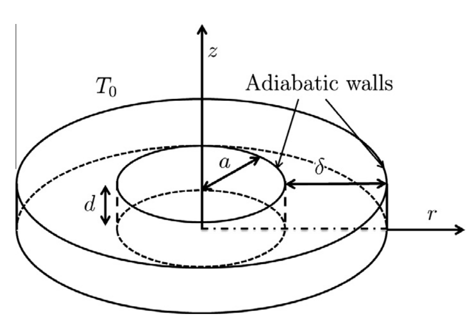

Fig. 1. Sketch of the geometry. Lateral walls are considered adiabatic. The fluid is heated from below and the top surface is open to the atmosphere.