International Journal of Heat and Mass Transfer 73 (2014) 97–102

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Thermal radiation analysis of packed bed by a homogenization method

Yusuke Asakuma [a,*], Yushin Kanazawa [a], Tsuyoshi Yamamoto [b,1]

[a] Department of Mechanical and Systems Engineering, University of Hyogo, Shosha 2167, Himeji, Hyogo 671-2280, Japan
[b] Department of Chemical Engineering, Kyushu University, 744 Motooka, Nishi-ku, Fukuoka 819-0395, Japan

ARTICLE INFO

Article history:
Received 16 October 2013
Received in revised form 27 January 2014
Accepted 27 January 2014
Available online 25 February 2014

Keywords:
Effective thermal conductivity
Homogenization method
Multiscale analysis
Microstructure
Thermal radiation

ABSTRACT

Effective thermal conductivity with radiation is analyzed by the homogenization method. This method can precisely represent the microstructure of a packed bed. In this study, the effects of parameters such as the radiation emissivity, temperature and particle size of the packed bed on the conductivity have been estimated to clarify the mechanism of complex packed structure. For example, heat transfer by radiation does not dominate if the material has voids of less than 1 mm in size. Moreover, by comparing a conventional model and the homogenization method, applicability of their models were shown for estimating the effective thermal conductivity.

1. Introduction

Packed bed reactors have been used in environmental processes such as reducing harmful exhaust (e.g. NO_x and SO_x) and producing new energy sources (e.g. generating hydrogen from methane). In the case of a catalytic reactor, for example, small particles made from a material such as alumina are tightly packed to achieve a large surface area. However, as the particle size is reduced, the superficial velocity decreases, which prevents efficient operation. Moreover, if the reaction is endothermic and heat must be added, heat transfer will be the most important factor of a bed. Accordingly, heat transfer in the bed must be understood and controlled to enhance performance.

Mass transfer and heat transfer in a bed are highly complex. For example, bed temperatures associated with steam methane reforming typically range from 700 to 1100 °C [1]. Since the reaction is highly endothermic, it is driven by heat conduction from the wall or preheated gas. At higher temperatures, thermal radiation must be considered apart from heat conduction and convection. Although conventional and empirical models have been proposed for considering the various behaviors [2,3], more precise thermal analysis is required to understand heat transfer due to thermal radiation in a bed. The homogenization method, developed using numerical theory, is proposed in the current study for performing thermal analysis of packed beds, and is considered to be highly useful from the viewpoint that it evaluates precise changes in microstructure and temperature by employing a three-dimensional finite element method. Owing to these beneficial features, this method has been used for structural analysis [4,5] and heat transfer analysis of composites and fiber [6–10]. In this study, thermal radiation is added to an existing packed bed model [11,12], and the effective thermal conductivity (ETC) is calculated in order to study the characteristic behavior of the packed bed during heat transfer at high temperatures.

2. Model

2.1. Homogenization method

To analyze the packed bed in Fig. 1(a), the simple periodic composite structure in Fig. 1(b) is considered. Each cell of this periodic structure consists of two domains: solid, Ω_s, and gas, Ω_g, as shown in Fig. 1(c). In what follows, subscripts s and g denotes the solid and gas components, respectively, and Γ denotes the interface between their two domains.

The periodic domain Ω is small compared with the characteristic length L at the macroscopic scale:

$$\varepsilon = \frac{l}{L} << 1, \qquad (1)$$

where ε is a scale parameter, and l and L can be understood as the characteristic sizes of the sample at the microscopic and the macroscopic scales, respectively. In this analysis, l is the particle diameter of the packed bed and ε ranges from about 1×10^{-6} to 1×10^{-4}.

* Corresponding author. Tel.: +81 79 267 4847.
E-mail addresses: asakuma@eng.u-hyogo.ac.jp (Y. Asakuma), yamamoto@chem-eng.kyushu-u.ac.jp (T. Yamamoto).
[1] Tel.: +81 92 802 2754.

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.01.065
0017-9310/

Nomenclature

A	finite area [m^2]
Bi	Biot number [–]
dA	diffuse differential area [m^2]
F	geometric configuration factor between two surfaces [–]
g	volumetric rate of heat generation [W/m^3]
h	interfacial thermal conductance [W/(m^2 K)]
h_r	heat transfer coefficient of radiation among particle [W/(m^2 K)]
L	characteristic macroscopic length [m]
l	characteristic microscopic length [m]
q_{rad}	heat flux of radiation [W/m^2]
$\boldsymbol{n}$	unit normal to Γ
r	distance between two surfaces [m]
T	dimensional Temperature [K or °C]
ΔT	imposed temperature difference [K]
x	dimensionless macro-scale variable [–]
x'	dimensional macro-scale variable [m]
y	dimensionless microscale variable [–]
α	ratio of thermal conductivity [–]
χ	particular solution for T [–]
δ	identity matrix
ε	scale parameter (l/L) [–]
ε_r	emissivity [–]
ϕ	angle between normal and line between two surfaces [–]
Γ	common boundary of the two media
Λ	dimensionless thermal conductivity [–]
λ	dimensional thermal conductivity [W/(m K)]
σ	Stefan–Boltzmann constant [W/(m^2 K^4)]
v	weight function [K]
θ	dimensionless temperature [K]
Ω	domain

Subscripts

eff	effective
g	gas phase
p	number of spatial dimension
q	number of spatial dimension
s	solid phase
sur	surroundings
0, 1, 2	asymptotic expansion indices in Eqs. (16)–(23)
1, 2	Surface in Eqs. (29)-(31)

The multiscale periodic heat conduction problem under steady-state conditions for the medium described above can hence be mathematically expressed as

$$-\frac{\partial}{\partial x_j^*}\left(\lambda_s \frac{\partial T_s}{\partial x_j^*}\right) = g_s \quad \text{in} \quad \Omega_s, \tag{2}$$

$$-\frac{\partial}{\partial x_j^*}\left(\lambda_g \frac{\partial T_g}{\partial x_j^*}\right) = g_g \quad \text{in} \quad \Omega_g, \tag{3}$$

$$-\lambda_s \frac{\partial T_s}{\partial x_j^*}\boldsymbol{n}_j = -\lambda_g \frac{\partial T_g}{\partial x_j^*}\boldsymbol{n}_j \quad \text{on} \quad \Gamma, \tag{4}$$

$$-\lambda_s \frac{\partial T_s}{\partial x_j^*}\boldsymbol{n}_j = h(T_s - T_g) \quad \text{or} \quad = q_{\mathrm{rad}} \quad (\text{See Eq.}(25)) \quad \text{on} \quad \Gamma, \tag{5}$$

where λ, T and g are the thermal conductivity, temperature field and volumetric rate of heat generation on a microscopic scale, respectively. Furthermore, $\boldsymbol{n}$ is the outward-pointing unit vector locally normal to the boundary Γ, and h is the interfacial thermal conductance. In case of radiation analysis, q_{rad} is used. Eqs. (2)–(5) are general expressions, and g_s and g_g become zero in the case that the bed is packed.

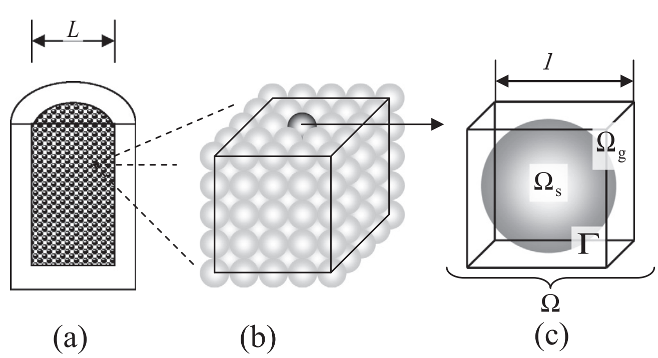

Fig. 1. Schematic diagram of homogenization method. (a) Packed bed (b) Periodic structure (c) Unit cell.

By defining the following nondimensionalized quantities,

$$y \equiv \frac{x^*}{l}, \quad \theta \equiv \frac{T}{\Delta T}, \quad \Lambda \equiv \frac{\lambda_g}{\lambda_s}, \tag{6}$$

in which ΔT is the external temperature difference on the macroscopic scale, we can rewrite Eqs. (2)–(5) as

$$-\frac{\partial}{\partial y_j}\left(\frac{\partial \theta_s}{\partial y_j}\right) = 0 \quad \text{in} \quad \Omega_s, \tag{7}$$

$$-\frac{\partial}{\partial y_j}\left(\Lambda \frac{\partial \theta_g}{\partial y_j}\right) = 0 \quad \text{in} \quad \Omega_g, \tag{8}$$

$$-\frac{\partial \theta_s}{\partial y_j}\boldsymbol{n}_j = -\Lambda \frac{\partial \theta_g}{\partial y_j}\boldsymbol{n}_j \quad \text{on} \quad \Gamma, \tag{9}$$

$$-\frac{\partial \theta_s}{\partial y_j}\mathbf{n}_j = \mathrm{Bi}(\theta_s - \theta_g) \quad \text{on} \quad \Gamma. \tag{10}$$

Here, the dimensionless heat generation numbers and the Biot number of convection and radiation with radiation heat transfer coefficient, h_r are given by

$$\mathrm{Bi} \equiv \frac{hl}{\lambda_s} \quad \text{or} \quad \mathrm{Bi} \equiv \frac{h_r l}{\lambda_s}. \tag{11}$$

Multiplying Eqs. (7) and (8) by a weight function v, integrating over Ω and applying Green's first identity theorem we obtain

$$\int_{\Omega_s} \frac{\partial v_s}{\partial y_j}\frac{\partial \theta_s}{\partial y_j}dy - \int_\Gamma v_s \frac{\partial \theta_s}{\partial y_j}\boldsymbol{n}_j ds = 0, \tag{12}$$

$$\int_{\Omega_g} \Lambda \frac{\partial v_g}{\partial y_j}\frac{\partial \theta_g}{\partial y_j}dy + \int_\Gamma \Lambda v_g \frac{\partial \theta_g}{\partial y_j}\boldsymbol{n}_j ds = 0 \tag{13}$$

and we substitute Eqs. (9) and (10) into Eqs. (12) and (13):

$$\int_\Omega \alpha \frac{\partial v}{\partial y_j}\frac{\partial \theta}{\partial y_j}dy - \int_\Gamma \mathrm{Bi}\, v\, \theta\, ds = 0, \tag{14}$$

where $\alpha = 1$ if $y \in \Omega_s$ and $\alpha = \Lambda$ if $y \in \Omega_g$.