



Analytical solution for combined heat and mass transfer in laminar falling film absorption using first type boundary conditions at the interface



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In memoriam Prof. Dr.-Ing. Dr.-Ing. h.c. Hans Dieter Baehr (1928–2014).

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ABSTRACT

Since the late seventies of the 20th century, several analytical models for combined heat and mass transfer in laminar falling film absorption have been proposed. Nevertheless the analytical solutions obtained with the Fourier method for the coupled process are complex and for short flow length a certain instability occurs which have been explained with the inconsistency of the initial and boundary conditions. Therefore boundary layer models have been justified in order to solve the transfer problem for short flow length. Moreover a linear approximation of the phase equilibrium is required.

The analytical solutions for heat and mass transfer presented in this paper are obtained by using the Laplace transform to solve the partial differential equations for an isothermal as well as impermeable wall. An originally unknown constant temperature and mass fraction boundary condition at the interface are set. The temperature and mass fraction profile across the film are obtained formally independently. In order to determine the yet unknown interface temperature and mass fraction the phase equilibrium and the interface energy balance are applied, using averaged gradients with regard to the streamwise coordinate. The interface temperature and mass fraction obtained with this procedure are interpreted and treated as mean values. From the known evolution of the mean interface temperature and mass fraction, the local values are derived by inverting the first mean value theorem for integration. The results show very good agreement to the established analytical solutions. The solving procedure does not depend on the input parameters such as the Lewis number for instance, which is needed in order to determine the eigenvalues within the Fourier method. Moreover arbitrary correlations for the phase equilibrium are applicable. The present solution is mathematically stable and offer explicit expressions in order to calculate the mean heat and mass fluxes directly. Therefore this solution is favourable to implement entire absorption process simulation, yet describing the coupled heat and mass transfer process comprehensively.

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1. Introduction

Absorption heat transformation devices are one option to increase the energy system's efficiency by using waste heat to provide valuable energies. The economic competitiveness of such devices is strongly dependent on the effectiveness of the absorber and desorber. Analyzing the coupled mass and energy transport phenomena during the absorption process therefore is an important issue. Simple and easily implementable but nevertheless realistic models are necessary in order to determine the crucial parameters influencing the absorption process.

2. State of the art

Nakoryakov and Grigor'eva [1] presented analytical solutions for combined heat and mass transfer using a boundary layer approach for both temperature and mass fraction profile within drops of absorbing solution. They also considered vapour absorption of a film flowing over an impermeable, isothermal wall. By evaluating the development of the temperature profile for the film flow, they found the temperature profile to be almost linear for the Lewis numbers considered [1]. Nakoryakov and Grigor'eva also used a boundary layer approach to investigate the heat and mass transfer for the entrance region of a falling film [2,3].

As the first exact analytical solution for combined heat and mass transfer for the whole film flow on a vertical wall, Grigor'eva

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Nomenclature

Dimensionless numbers Le Lewis number ($Le = a/D$)
 \tilde{St} modified Stefan number ($\tilde{St} = c_s \cdot \Delta T / (\Delta h_{abs} \cdot \Delta c)$)

Greek letters

α, β eigenvalues
 Δ difference
 δ film thickness (m)
 η dimensionless film thickness (6)
 γ dimensionless absorbate mass fraction (9)
 λ thermal conductivity ($W (m K)^{-1}$)
 μ dimensionless mass flux (58)
 ρ density ($kg m^{-3}$)
 σ dimensionless accumulated mass (60)
 Θ dimensionless temperature (8)
 ξ normalised flow coordinate (7)

Latin letters

\dot{m} mass flux ($kg m^{-2} s^{-1}$)
 \dot{q} heat flux ($kJ m^{-2} s^{-1}$)
 a thermal diffusivity ($m^2 s^{-1}$)
 A, B constants
 c mass fraction (absorbate) ($kg kg^{-1}$)
 c specific heat capacity ($kJ (kg K)^{-1}$)
 D mass diffusivity ($m^2 s^{-1}$)

h specific enthalpy ($kJ kg^{-1}$)
 i imaginary unit
 k running index
 m accumulated mass per area ($kg m^{-2}$)
 p pressure (Pa)
 s complex Laplace variable
 T temperature (K)
 u streamwise film velocity ($m s^{-1}$)
 x streamwise direction (m)
 x mass fraction (absorbent) ($kg kg^{-1}$)
 y transverse direction to film flow (m)

Sub-/Superscripts/Symbols

0 initial values
 $\circ \bullet$ Doetsch (correspondence) symbol
 $\overline{(\)}$ mean value
 $(\)$ modified
 abs absorption
 eq equilibrium (at inlet condition)
 i interface
 k running index
 $LiBr$ lithium bromide
 s solution
 W wall

and Nakoryakov presented series of eigenfunctions using the Fourier method [4].

They applied an isothermal and impermeable wall boundary condition as well as phase equilibrium and energy conservation as interface boundary conditions. A uniform film velocity is applied and the phase equilibrium condition at the interface is assumed to be linear for a given pressure:

$$T_i = B - A \cdot c_i. \quad (1)$$

Nakoryakov et al. [5] reported oscillations for small dimensionless flow length ξ due to the inconsistency of the initial and boundary conditions for the solution obtained with the Fourier method and therefore justified the necessity of their short term solution [2]. Nevertheless the range of validity of both, the Fourier as well as the short term solution in dependency of varying initial temperatures and dimensionless numbers have not been presented by Nakoryakov et al.

Grossman [6] extended this model, taking into account the Nusselt [7] parabolic film velocity profile. Grossman also used the Fourier method solving the partial differential equations for heat and mass transfer analytically. He stated that the obtained analytical solution is accurate for moderate and large values of the normalised coordinate ξ in flow direction. For small values of ξ , Grossman has used numerical methods combined with a short time analytical solution which is only valid for the unaffected boundary layer development similar to the solution in [2].

Le Goff [8] introduced the Laplace transform to solve the coupled partial differential equations for heat and mass transfer. However, due to difficulties in applying the inverse Laplace transform to the solutions obtained for the coupled process in the s -domain, the Fast Fourier Transform has been used to invert the solutions from the s -domain back to the time domain.

As the mass transfer resistance in the liquid phase is expected to dominate the transfer process, Conlisk [9] assumed the mass transfer taking place only in a thin layer close to the interface. The Laplace transform has been applied to the partial differential

equation for this mass transfer problem and an error function solution in the time domain was obtained, assuming the temperature profile to be developed similar to [1]. In [10] Conlisk and Mao extended the former Conlisk model [9] by transient heat transfer using Fourier cosine transform.

A simple model for falling film absorption of water into aqueous solutions of lithium bromide has been presented by Wohlfeil [11] and Auracher et al. [12], assuming transient mass transfer and the temperature profile to be developed, similar to [9].

The objective of this paper is to merge the simplicity of the model presented by Wohlfeil [11] and the accuracy of the complex model presented by Grigor'eva and Nakoryakov [1,5]. In addition, a way of how to implement non-linear relationships between composition and boiling temperature is presented.

3. Modelling and assumptions

According to the left hand side of Fig. 1 laminar falling film absorption on a horizontal tube is considered. Droplets of solution impinge on the top of the tube, spread and form a film flow on the tube's surface. The solution merges at the bottom of the tube forming droplets again that detach from the tube. All processes concerning droplets are disregarded and only the film flow on the tube's surface is considered. According to the right hand side of Fig. 1 the curvature of the film on the tube is neglected and modelled as plane geometry.

The governing differential equations in cartesian coordinates (2) and (3) are derived from the differential energy and mass balances taking into account convective transport in streamwise direction x and diffusive transport in transverse direction y only. A constant mean film velocity \bar{u} is assumed:

$$\bar{u} \frac{\partial T}{\partial x} = a \frac{\partial^2 T}{\partial y^2}, \quad (2)$$

$$\bar{u} \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial y^2}. \quad (3)$$

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