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Combined effects of uniform heat flux boundary conditions and hydrodynamic slip on entropy generation in a microchannel

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The effects of wall heat flux boundary conditions, wall to fluid thermal conductivity ratio and slip flow on heat transfer and entropy generation by considering the conjugate heat transfer problem in microchannels are studied, analytically. The heat transfer equations in the fluid and the finite thickness walls of the microchannel are solved analytically using uniform heat flux boundary conditions at the outer surfaces of the walls with appropriate continuity of temperature and heat flux at the fluid-wall interfaces. Exact analytic solutions for the velocity and temperature fields in the fluid and walls of microchannel are utilized to compute the entropy generation rate. The latter is integrated in the whole region of analysis so that the finite dimensions of the device are considered to get the global entropy generation rate. Finally, this quantity is discussed in detail and investigated considering combined effects of wall and hydrodynamic slip. Findings reveal that it is possible to find optimum values of heat flux across the walls of microchannel where the global entropy generation reaches a minimum. Special attention has been given to the effect of the wall heat flux on optimal values of other parameters. The optimum values of both the slip length and wall to fluid thermal conductivity ratio, where the entropy generation is minimum, decrease with the wall heat flux. Also, optimum values of Peclet number with minimal entropy are found for certain suitable combination of geometrical and physical parameters of the system.

1. Introduction

One of the important objectives of thermal systems engineering is to analyze the utilization of thermal energy in an efficient manner. From the viewpoint of thermodynamics, the decrease of entropy generation means the decrease of irreversibility and less loss of exergy. Therefore, in the energy optimization problems and design of many traditional heat removal engineering devices, it is necessary to evaluate the entropy generation or exergy destruction due to heat transfer and viscous friction as a function of the physical and geometrical parameters selected for the optimization analysis. This procedure, known as the Entropy Generation Minimization (EGM) method [1,2] is a thermodynamic approach of optimization of engineering systems for higher energy efficiency.

The EGM has become an useful optimization tool for a wide range of thermal applications. This method has already been applied to numerous devices and processes, and many papers and books on this topic have been published. The search for conditions that lead to the minimization of entropy generation in a given macro-scale process or device has been the task of several investigations reported in the literature [3–17], such as heat exchangers [3,4], two-phase flows [5], magnetohydrodynamic (MHD) flows subjected to ohmic dissipation [6–10], heat transfer problems with temperature dependent heat sources [14], flows with temperature dependent fluid viscosity [16,17], among many others. However, previous experimental, analytical and numerical studies have identified the importance of entropy production in optimal designs of microfluidic devices [18–26]. Reducing entropy generation of fluid motion through microchannels can have a beneficial impact on the input power required to achieve both desired heat exchange and mass flow rates. Recently, some works have addressed the analysis of entropy generation in microdevices considering mainly the effects of viscous and thermal irreversibilities [18–21]. Haddad et al. [18] investigated the entropy generation due to steady laminar forced convection fluid flow through parallel plates microchannel, numerically. They found that the entropy generation within the microchannel decreases as Knudsen number increases, and increases as Reynolds, Prandtl, Eckert numbers and the nondimensional temperature difference increase. Also, the contribution of

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the viscous dissipation in the total entropy generation increases as Knudsen number increases over wide ranges of the flow controlling parameters. Chen et al. [19] calculated the entropy generation in microchannel flows and analyzed for different thermal boundary conditions. Local entropy generation rate was found to be only dependent on the temperature gradient in the flow direction. Abbassi et al. [20] analyzed the issue of entropy generation in a uniformly heated microchannel heat sink. They used an analytical approach to solve the forced convection problem across the microchannel using a porous medium model based on extended Darcy equation for fluid flow and two-equation model for heat transfer. They found an optimized value for porosity at which entropy generation rate reaches its minimum magnitude. Erbay et al. [21] investigated the entropy generation in microchannels induced by the transient laminar forced convection in the entrance region between two parallel plates, numerically. They found that the entropy generation has its highest value at the highest Reynolds and Prandtl values considered in the study. In turn, the dissipative processes that arise in a microchannel flow subjected to electromagnetic interactions have been analyzed in [22–26], where the contribution of thermal, friction and electromagnetic irreversibilities to the total rate of exergy destruction in these devices has been assessed. On the other hand, entropy generation analysis at microscale with the inclusion of hydrodynamic slip is also reported by various authors [27–31]. For instance, Ogedengbe et al. [27] have investigated the mechanisms of near-wall velocity slip and their effects on energy conversion of fluid motion in microchannels. They found that various design parameters (such as operating pressure and channel aspect ratios) can be modified to reduce entropy generation in microchannels for gas flows. In another task [28], they have performed a numerical study of slip flow irreversibility effects in a counterflow heated microchannel. The effects of channel size perturbation, Reynolds number, and pressure ratios on the exergy destruction are presented. Hooman [29] has studied the entropy generation for forced convection in microelectromechanical system in the slip flow regime. It was found that dimensionless entropy generation number, Ns, always decreases with Knudsen number, Kn. Yazdi et al. [30] have analyzed the entropy generation in external liquid flow over a surface containing embedded parallel microchannels. The results showed that the rate of entropy generation always decreases with increasing slip length. Ibáñez et al. [31] have analyzed the effects of slip flow on heat transfer and entropy generation by considering the conjugate heat transfer problem in microchannels using thermal boundary conditions of the third kind at the outer surfaces of the walls. They found an optimum slip flow where the entropy generation is minimum. The above studies have shown that a fundamental understanding of the fluid flow and heat transfer in microchannels is of great importance for achieving optimum design of the devices. Although the entropy generation rate is calculated in these microfluidic processes and devices, analytical solutions for the conjugate heat transfer problem with heat flux boundary conditions in both the lower and upper walls, simultaneously; and their effects on entropy generation have not been analyzed. Moreover, in many of these studies, the thermal analysis is restricted to constant temperature boundary conditions and cases where slip flow and convective effects can be disregarded.

Therefore, the present analysis differs from the aforementioned studies in that the conjugate heat transfer problem in the walls of microchannel and in the ordinary hydrodynamic microfluid is solved analytically with uniform heat flux boundary conditions at the outer surfaces of both the top and bottom walls by considering slip flow and convective heat transfer, simultaneously. Hence, our interest is mainly addressed to explore the influence of the wall heat flux boundary conditions on heat transfer and entropy generation, and to show the existence of optimal values of this quantity in the microdevice consistent with minimum entropy generation rate. It is noted that the present solution extends the work of Ibáñez et al. [31] to include the combined effects of wall heat flux boundary conditions, slip flow and convective heat transfer on entropy generation using an analytical solution for the velocity and temperature fields. Besides, the solution procedure used in the present analysis differs from the method used in [31], it differs in that now in addition to the six equations obtained by evaluating the six boundary conditions of the mathematical model, in the solution procedure an additional energy balance equation is required to obtain the constants and solve the conjugate heat transfer problem for the temperature.

The above shows that the fundamental contributions of the work are: a) The entropy production is determined from the analytical solution of the conjugate heat transfer problem combining wall heat flux boundary conditions with slip flow and convective heat transfer, b) Relevant dimensionless parameters for the system are optimized by minimizing the global entropy generation. Optimum values of wall heat flux, wall to fluid thermal conductivity ratio, Peclet number and slip flow that minimize the entropy generation are found, c) The effects of heat flux across the walls on the optimum values of some other parameters are also investigated.

In the following sections, the problem is formulated, analyzed, solved and discussed. Section 2 consists of the transport problem analysis which contains the momentum and energy balance equations and their solutions. Section 3 contains the determination of the entropy generation rate and graphical representation of results and their discussion. Section 4 contains the concluding remarks.

2. Transport problem

2.1. Velocity field

We consider the steady fully developed flow of a viscous fluid between two infinite parallel walls of finite thickness separated by a distance $2a$ in the presence of a longitudinal constant pressure gradient, dp/dx. In this way, the only velocity component is in the axial x-direction and depends on the transversal y-coordinate The upper wall is located at $y' = a$ and the lower wall is at $y' = -a, y'$ denoting the dimensional transversal coordinate. We also assume that the fluid is incompressible and monocomponent, so that the mass diffusion phenomenon is disregarded. At the fluid-wall interfaces, the slip conditions on the velocity are applied. Under these conditions, from the solution of the momentum balance equation we find that the velocity profile in dimensionless form is [31]

$$u = -\frac{3}{3F_1 - 1}\left[F_1 + \frac{2(\alpha_2 - \alpha_1)}{2 + \alpha_1 + \alpha_2} y + y^2\right], \tag{1}$$

where

$$F_1 = \frac{3(\alpha_1 + \alpha_2) + 4(\alpha_1 \alpha_2) + 2}{2 + \alpha_1 + \alpha_2},$$

where the dimensionless transversal coordinate y, lower wall slip length α_1 and upper wall slip length α_2 are normalized by half the distance between the channel walls, a. In turn, the velocity, u, has been normalized by the average velocity in the cross section of the microchannel U, given by

$$U = \frac{1}{2a}\int_{-a}^{+a} u' dy' = -\frac{a^2}{2\eta}\left(\frac{dp}{dx'}\right)\left[\frac{3(\alpha_1' + \alpha_2') + \frac{4}{a}(\alpha_1' \alpha_2') + 2a}{2a + \alpha_1' + \alpha_2'} - \frac{1}{3}\right].$$

Here, u' is the dimensional velocity, η is the dynamic viscosity of the fluid and x' is the dimensional longitudinal coordinate, while α_1' and α_2' are the dimensional slip lengths of the lower and upper walls, respectively.