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Conjugate natural convection combined with surface thermal radiation in a three-dimensional enclosure with a heat source

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ABSTRACT

Numerical analysis of natural convection and surface thermal radiation in a cubical cavity having heat-conducting solid walls of finite thickness with a heat source located at the bottom of the cavity in conditions of convective heat exchange with an environment has been carried out. Mathematical study has been based on a numerical solution of the three-dimensional Boussinesq equations in the dimensionless variables such as vector potential functions, vorticity vector and temperature by finite difference method. Main attention was paid to the effects of the Rayleigh number $10^3 \leqslant Ra \leqslant 10^5$, an emissivity of internal surfaces of the solid walls $0 \leqslant \varepsilon < 1$, a thermal conductivity ratio $1 \leqslant k_{1,2} \leqslant 15$ and a dimensionless time $0 \leqslant \tau \leqslant 100$ on the velocity and temperature fields. The effect scales of key parameters on the average Nusselt numbers have been determined.

1. Introduction

The problem of natural convective heat transfer in enclosures has been extensively studied due to its importance in many practical applications such as the design of solar collectors, energy efficient buildings, cooling of electronic equipment, and thermal behavior in reactors. It is well known [1–4] that when natural convection in air or inert gases is involved, the heat transfer by convection and radiation are usually of the same order of magnitude. Although in most practical cases the flow is three-dimensional and heat conduction plays an important role due to this heat transfer mechanism can lead to a variation of the average Nusselt number up to 45% [5]. There are some papers concerning a combined analysis of natural convection and thermal radiation in three-dimensional enclosures [6–15]. For example, Borjini et al. [6] have numerically studied the effect of the radiative heat transfer on the three-dimensional convection of a cubic radiatively participating melt with $Pr = 13.6$ and $Ra = 10^5$. The medium is considered as a gray, emitting–absorbing and isotropically scattering fluid. It was found that the effect of the radiative heat transfer on the 3D behavior of the flow is significant in the core of the enclosure. The inner spiraling flows are very sensible in location and direction to the radiative heat transfer while the peripheral spiraling motion is qualitatively intensive to this mode. Colomer et al. [7] have investigated the phenomenon of radiation and natural convection in both transparent and participating media in a three-dimensional differently heated cavity. It was shown that in a transparent medium the radiation significantly increases the heat flux and that for a given Planck number and constant reference temperature ratio the contribution of radiation remains almost constant for a range of Rayleigh numbers. In case of participating media, defined by its optical thickness, the heat flux both increases with the Rayleigh number and decreases with the optical thickness. Albanakis and Bouris [8] have conducted a three-dimensional numerical analysis of combined heat transfer (turbulent mixed convection, surface radiation and conduction) for an asymmetrically heated cube exposed to turbulent external flow. They have found that radiation effects between the inner surfaces of a cubic enclosure can play a significant role in determining the temperature distribution. Xin et al. [9] by means of DNS using spectral methods have numerically studied coupled heat transfer (natural convection and radiation in an air-filled cubical cavity and heat conduction in solid walls). It was found that surface radiation is an important factor that affects natural convection in air-filled cavities and that the experimental results of natural convection in air have resulted from the coupled phenomena of convection, conduction and radiation. Surface

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Nomenclature

A_1, A_2 surface element
$Bi = hL/k_1$ Biot number
F_{k-i} view factor from kth element to the ith element of an enclosure
g acceleration of gravity
G auxiliary function
h heat-transfer coefficient
$k_{1,2} = k_1/k_2$ thermal conductivity ratio
k_1 thermal conductivity of the wall material
k_2 air thermal conductivity
l thickness of walls
L size of an air cavity
$\mathbf{n}$ unit normal vector
$N_{rad} = \sigma T_{hs}^4 L/[k_2(T_{hs} - T^e)]$ radiation number
$\overline{Nu}_{con} = \int_{0.1}^{1.1}\int_{0.1}^{1.1}\left|\frac{\partial\Theta}{\partial Z}\right|_{Z=0.14} dXdY$ average convective Nusselt number
$\overline{Nu}_{rad} = N_{rad}\int_{0.1}^{1.1}\int_{0.1}^{1.1} Q_{rad}\Big|_{Z=0.14} dXdY$ average radiative Nusselt number
NS number of surface elements in an enclosure
$Pr = \nu/\alpha_2$ Prandtl number
Q_{rad} dimensionless net radiative heat flux
R_k dimensionless radiosity of the kth element of an enclosure
$Ra = g\beta(T_{hs} - T^e)L^3/\nu\alpha_2$ Rayleigh number
S distance between two surface elements
t time
T temperature
T^e environmental temperature
T_{hs} heat source temperature
u, v, w velocity components in x, y, z directions
U, V, W dimensionless velocity components in X, Y, Z directions
x, y, z Cartesian coordinates
X, Y, Z dimensionless Cartesian coordinates
$\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}$ dimensionless Laplacian

Greek symbols
$\alpha_{1,2} = \alpha_1/\alpha_2$ thermal diffusivity ratio
α_1 thermal diffusivity of the wall material
α_2 air thermal diffusivity
β coefficient of volumetric thermal expansion
γ angle
$\Delta\tau$ computational time step
ε surface emissivity
$\varsigma = T^e/T_{hs}$ temperature parameter
η, ξ Cartesian coordinates
Θ dimensionless temperature
ν kinematic viscosity
Ξ dimensionless stream function
σ Stefan-Boltzmann constant
τ dimensionless time
ψ_x, ψ_y, ψ_z dimensional vector potential functions
Ψ_x, Ψ_y, Ψ_z dimensionless vector potential functions
$\omega_x, \omega_y, \omega_z$ dimensional vorticity vector components
$\Omega_x, \Omega_y, \Omega_z$ dimensionless vorticity vector components

Subscripts
con convective
e environment
hs heat source
max maximum
rad radiative

radiation reduces the thermal stratification through not only the horizontal walls but also the front and rear walls. Fusegi et al. [10] have conducted 3D finite-difference numerical study of natural convection and gas radiation in a differently heated cubical enclosure. A gas radiation model used in the analysis was based on the P_1-differential approximation method [11]. They found that radiation enhances the three-dimensionalities of the velocity and temperature fields and an increase in the overall temperature of the insulated plates is mainly attributed to radiation effects. Gossard and Lartigue [12] have conducted numerical and experimental analysis of natural convection, thermal radiation and heat conduction in three-dimensional building components with air-filled vertical cavities. They shown that an inverse method using a particle swarm optimization algorithm is effective for the considered problem and allows to find several suitable solutions differentiated by their thermophysical and geometrical properties. Menguc and Viskanta [13,14] have studied radiative heat transfer in a three-dimensional rectangular enclosure containing radiatively participating gases and particles using P_1- and P_3-approximations. Kuznetsov and Sheremet [15] have numerically analyzed 3D conjugate natural convection and radiation on the basis of the Rosseland approximation in enclosures. It has been shown that the average Nusselt number is an increasing function of the thermal conductivity ratio and the Rayleigh number and a decreasing function of the optical thickness of the medium.

From the above literature survey it is evident that transient conjugate natural convection with surface radiation in a three-dimensional enclosure with a heat source in a heat exchange with an environment has not been studied in detail.

The objective of the present work is a numerical simulation of transient combined heat transfer (natural convection, radiation and conduction) in a cubical enclosure having heat-conducting solid walls of finite thickness and a heat source located at the bottom of the cavity in conditions of convective heat exchange with an environment. Comparison of the obtained results with data for 2D model [16] allows to analyze an effect of third coordinate.

The obtained results have fundamental and applied values. The former is related to the development of the conjugate heat transfer theory. Here it should be noted that in the case of conjugate natural convection an effect of surface radiation is more pronounced [9,16] due to physical mechanisms of radiation inside the transparent medium. As well known firstly thermal radiation affects the solid walls and as a result the solid walls temperature increases that leads to an intensive air motion close to the solid wall surfaces. This intensive motion due to the convective heat transfer leads to changes of thermo-hydrodynamic parameters. If we do not take into account the finite thickness and conductivity of solid walls it can lead to essential differences between the obtained results and physical data [9].

As for engineering application the obtained results are related to cooling of electronic components. At the present time, an evolution of the elementary potential electronic components is related to usage of high power in rather small volumes. Such tendency leads to sharp increase in a specific dissipated power and hence in dispersed heat density. Therefore development of methods of heat removal and temperature control inside a product takes on special significance at designing of electronics [17–20]. It should be noted that an increase in temperature of microchips and