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Mixed convection in a vertical helical annular pipe

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ABSTRACT

Mixed convection in a vertical helical annular pipe is studied numerically using a second order finite difference method based on the projection algorithm. To do so, the governing equations are written in the helical orthogonal coordinate system and discretized on a uniform three-dimensional grid. Considering developing flow and the Boussinesq approximation for the buoyancy term in the momentum equation, mixed convection inside a helical annular pipe is investigated by applying a constant heat flux on the inner pipe wall while keeping the outer pipe wall insulated. The effects of governing non-dimensional parameters such as torsion, curvature, Reynolds number, Prandtl number, and Richardson number on the flow and temperature field patterns and consequently on the friction factor and heat transfer rate are studied in detail.

1. Introduction

Mixed convection in a vertical helical annular pipe is of interest in many engineering applications such as heat exchangers, chemical reactors, cooling channels and lubrication systems. In addition, some natural systems benefits from this kind of system such as blood vessels and water duct in plants. Hence, the study of fluid flow behavior and its thermal characteristics help engineers design more efficient systems.

Various researchers have carried out different studies on the fluid flow and heat transfer in curved and helical pipes. In an early work, Murata et al. [1] represent a two dimensional model to analyze the flow in a periodically curved pipe. Nobari and Mehrabani [2] investigated the fluid flow and heat transfer in an eccentric curved annulus. They concluded that the Nusselt number can be increased at large values of Dean Number. The study by Nobari et al. [3] on the flow in an eccentric curved pipe shows that decreasing the curvature radius moves the peak axial velocity and its sharp gradient towards the outer bend region due to the larger centrifugal forces. The continuity and Navier–Stokes equations were derived in a non-orthogonal coordinate by Wang [4] who surveyed the effect of torsion on the secondary flow at low Reynolds numbers. Thereafter, Germano [5] has introduced a helical orthogonal coordinate system for deriving the governing equations. He [6] has also extended his work to a helical pipe with an elliptical cross section to investigate the effect of torsion on the secondary flow field. Vasudevaia and Rajalakshmi [7] verified the Germano's results using a numerical approach in the orthogonal helical coordinate system and determined the torsion effect order on the flow field. The study by Kao [8] has considered the fluid flow inside a helical annulus using the series expansion method. The effect of geometric parameters such as torsion and curvature at different Reynolds numbers on the flow and pressure field was investigated by Liu and Masliyah [9] who indicated that at a constant geometry decreasing the Dean number causes secondary flow field to become asymmetric. Moreover, they demonstrated that by increasing the torsion the peak axial velocity shifts towards the center of annuli. Yamamoto et al. [10] have studied the effect of torsion on the flow field and heat transfer and have indicated that the heat transfer rate increases as torsion increases. The experimental study of the friction factor in the helical pipe by Yamamoto et al. [11] indicates that the variation of friction factor in a helical pipe is similar to the one in the curved. Furthermore, Yamamoto et al. [12] have conducted a numerical and experimental study for visualization of flow field in a helical pipe. Ali [13,14] has performed experiments on the horizontal and vertical helical pipes to study the natural convection heat transfer. He indicated that the heat transfer coefficient is directly proportional by the heat flux and Rayleigh number. The study by Huttl et al. [15] employed a finite volume method to investigate the flow field in a helical pipe under the effects of non-dimensional parameters such as Reynolds and Dean numbers. Detailed study of the flow and pressure field was carried out by Palazoglu and Sandeep [16] who concluded that the axial velocity peak decreases as the Dean number increases. Fully developed forced convection in a helical tube with uniform

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Nomenclature

b	body force
B	dimensionless body force
$\vec{B}$	binormal vector
c_p	specific heat at constant pressure
D_h	hydraulic diameter, $2(r_o - r_i)$
g	physical gravitational acceleration
g_r	gravitational acceleration in r direction
g_θ	gravitational acceleration in θ direction
g_s	gravitational acceleration in s direction
Gr	Grashof number, $\beta g q' D_h^4/k\nu^2$
h_s	coordinate scale factor in s direction
H_s	dimensionless coordinate scale factor in s direction, h_s/D_h
k	thermal conductivity
$\vec{n}$	unit normal vector
N	radius ratio
$\vec{N}$	normal vector
Nu	average Nusselt number
p	pressure
P	dimensionless pressure
Pr	Prandtl number, $pr = \upsilon/\alpha$
P_s	helical pitch
q'	heat flux
r	radial coordinate component
R	dimensionless radial component, r/D_h
r_a	radius of curvature
r_i	inner pipe radius
r_o	outer pipe radius
Re	Reynolds number, $u_m D_h/\nu$
Ri	Richardson number, Gr/Re^2
s	longitudinal coordinate component
t	time
T	temperature
T_m	mean temperature
$\vec{T}$	tangential vector
u	velocity component in s direction
u_m	mean axial velocity
U	dimensionless velocity component in s direction, u/u_m
v	velocity component in η direction
V	dimensionless Velocity component in r direction, v/u_m
$\vec{V}$	dimensionless velocity vector
w	velocity component in z direction
W	dimensionless velocity component in θ direction, w/u_m
S	dimensionless length, s/D_h

Greek letters

α	thermal diffusion
β	volumetric coefficient of thermal expansion
θ	angular coordinate component
Θ	dimensionless temperature, $(T - T_e)/(q'D_h/k)$
Θ_m	mean dimensionless temperature
κ	curvature
μ	viscosity of fluid
ν	kinematic viscosity
ρ	density of fluid
ρ_e	density of fluid at ambient temperature
τ	torsion

Subscriptions

e	entrance
i	inner wall
h	helical
m	average value
O	outer wall
S	straight pipe

wall temperature has been studied analytically based on the minimum entropy generation principle by Shokouhmand and Salimpour [17]. They have revealed that the optimum Reynolds number decreases as the curvature increases. Further study on the flow field in helical pipes with a non-uniform curvature and torsion has been carried out by Gammack and Hydon [18]. Nobari and Malvandi [19] studied flow in a helical annular pipe under hydrodynamically fully developed assumption and investigated the effects of different physical parameters. They have shown that a decrease in the aspect ratio and torsion number leads to the increase of the friction factor at a given Dean number.

In this article mixed convection in a vertical helical annular pipe has been studied numerically using a second order finite difference method based on the projection algorithm for the first time based on our knowledge. The governing equations written in the helical orthogonal coordinate system are discretized on the uniform grid. Assuming a developing fluid flow inside a vertical helical annular pipe, mixed convection heat transfer is investigated at the constant heat flux thermal boundary on the inner pipe with an insulated outer pipe wall. The effects of the governing non-dimensional parameters such as the non-dimensional torsion, curvature, Richardson number, Prandtl number, and Reynolds number on the flow and temperature field, and on the friction factor and heat transfer rate are investigated in detail.

2. Governing equations

Here incompressible viscous fluid flow and mixed convection heat transfer inside a vertical helical annular pipe is considered as shown in Fig. 1(a), where the concentric annulus with outer radius r_o is twisted around a cylinder with the radius r_a. Since the annulus extends upwards with a constant pitch, P_s, the curvature and torsion of the helical pipe can be defined respectively as

$$\kappa' = \frac{r_a}{r_a^2 + P_s^2} \tag{1}$$

$$\tau' = \frac{P_s}{r_a^2 + P_s^2} \tag{2}$$

where κ' is the dimensional curvature and τ' is the dimensional torsion. Based on the mean axial velocity, u_m, hydraulic diameter, D_h, heat flux, q', inlet temperature, T_e, thermal conductivity, k, pressure, p, density, ρ, the thermal diffusion, α, and kinematic viscosity, ν, the following non-dimensional parameters can be defined

$$U = \frac{u}{u_m}, \quad V = \frac{v}{u_m}, \quad W = \frac{w}{u_m}, \quad R = \frac{r}{D_h}, \quad S = \frac{s}{D_h},$$
$$\Theta = \frac{T - T_e}{q'D_h/k}, \quad \kappa = D_h\kappa', \quad \tau = D_h\tau', \quad P = \frac{p}{\rho u_m^2/2} \quad Pr = \frac{\upsilon}{\alpha},$$
$$\mathrm{Re} = \frac{u_m D_h}{\nu}, \quad Gr = \frac{\beta g q' D_h^4}{k\nu^2}, \quad Ri = \frac{Gr}{\mathrm{Re}^2} \tag{3}$$

where U is the dimensionless velocity component in the s direction, V the dimensionless velocity component in the r direction, W the dimensionless velocity component in the θ direction. Also, R is the dimensionless r coordinate, S the dimensionless s coordinate, Θ the dimensionless temperature, κ the dimensionless curvature ratio, τ the dimensionless torsion, P the dimensionless pressure,