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A multiple-relaxation-time lattice Boltzmann model for convection heat transfer in porous media

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ABSTRACT

In this paper, a two-dimensional (2D) multiple-relaxation-time (MRT) lattice Boltzmann (LB) model is developed for simulating convection heat transfer in porous media at the representative elementary volume scale. In the model, a D2Q9 MRT-LB equation is adopted to simulate the flow field, while a D2Q5 MRT-LB equation is employed to simulate the temperature field. The generalized model is employed to model the momentum transfer, and the effect of the porous media is considered by introducing the porosity into the equilibrium moments, and adding a forcing term to the MRT-LB equation of the flow field in the moment space. The present MRT-LB model is validated by numerical simulations of several 2D convection problems in porous media. The present numerical results are in excellent agreement with the analytical solutions and/or the well-documented data reported in previous studies. It is also found that the present MRT-LB model shows better numerical stability than the LBGK model.

1. Introduction

The analysis of convection heat transfer in porous media has attracted considerable attention due to its importance in the related technological and engineering applications, such as geothermal energy systems, chemical catalytic reactors, crude oil extraction, solar power collectors, electronic device cooling, and contaminant transport in groundwater. In the past several decades, various traditional numerical methods, such as the finite volume method, the finite difference method, and the finite element method, have been used to study convection heat transfer in porous media. Comprehensive literature surveys of this subject have been given by Cheng [1], and Nield and Bejan [2]. In particular, the non-Darcy effects on convection heat transfer in fluid-saturated porous media have been investigated numerically by many researchers [3–6].

The lattice Boltzmann (LB) method, which evolves from the lattice-gas automata (LGA) method [7], has gained great success in modeling complex fluid flows and simulating complex physics in fluids due to its kinetic background [8–12]. As a mesoscopic method based on the kinetic equation, the LB method has some distinctive merits over the traditional numerical methods (see, e.g., Ref. [13]). Owing to its kinetic nature and distinctive computational feature, the LB method has been successfully applied to study fluid flows in porous media soon after its emergence [14]. The existing LB models for porous flows can be generally classified into two categories, i.e., the pore scale method [14–16] and the representative elementary volume (REV) scale method [17–21]. In the pore scale method [14–16], the standard LB model is used to simulate the fluid flows in the pores, and the interaction between the solid and fluid phases is realized by using the no-slip bounce-back rule. The detailed flow information of the pores can be obtained by this method, which can be utilized to investigate macroscopic relations. However, this method needs the detailed geometric information of the pores, and each pore needs several lattice nodes in simulations, so the computation domain is small because of the limited computer resources [19]. In the REV scale LB method [17,18], an additional term is added to the standard LB equation to consider the effect of the porous media based on some semiempirical models, such as the Darcy model, the Brinkman-extended Darcy model and the Forchheimer-extended Darcy model. Based on the so-called Brinkman–Forchheimer-extended Darcy model (also called the generalized model) [6] that overcomes some limitations of the Darcy model, Brinkman-extended Darcy model and Forchheimer-extended Darcy model, Guo and Zhao [19] proposed a generalized LB model for simulating isothermal incompressible porous flows. In this model, the porosity is included into the equilibrium distribution function, and a forcing term is added to the LB

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equation to account for the Darcy (linear) and Forchheimer (nonlinear) drag forces of the solid matrix. Subsequently, Guo and Zhao [20] extended the generalized LB model to incompressible thermal flows in fluid-saturated porous media by using the double-distribution-function (DDF) approach. In the literature [21], the reliability and the computational efficiency of the LB method in studying natural convection flow in porous media has been confirmed. As reported in Ref. [21], the LB method needs less computational time than the finite difference method to obtain the same accurate solutions of natural convection flow in porous media on the same grid size.

However, to the best of our knowledge, the existing LB models for porous flows at the REV scale employ the Bhatnagar–Gross–Krook (BGK) collision model [22] to represent the collision process in the evolution equation. Although the lattice Bhatnagar–Gross–Krook (LBGK) model has become the most popular one for its extreme simplicity, it has also received several well-known criticisms, among which the most important one is the numerical instability at low viscosities. Fortunately, it has been demonstrated that the shortcomings of the LBGK model can be resolved by using the multiple-relaxation-time (MRT) collision model [23,24]. In the LB community, it has been widely accepted that the MRT model can improve the numerical stability by separating the relaxation rates of the hydrodynamic (conserved) and kinetic (non-conserved) moments [25–27]. In recent years, the MRT-LB method has been successfully applied to simulate complex fluid flows such as axisymmetric flows [28], microscale flows [29], and multiphase flows [30,31]. In Refs. [32–35], the MRT-LB method has been used to study convective flows in the absence of porous media. The basic idea of the MRT-LB method [32–35] is that the flow and temperature fields are solved separately by two different MRT-LB equations. As reported in Ref. [34], the MRT-LB model with two MRT-LB equations is second-order accurate for simulating incompressible thermal flows with the Boussinesq approximation. Recently, the double MRT-LB method with the large eddy simulation approach has been used to simulate the 3D heat transfer in a turbulent channel flow [36].

In this paper, we aim to develop a MRT-LB model for simulating convection heat transfer in porous media at the REV scale based on the generalized model, which can be viewed as an extension to some previous studies. Moreover, the stability of the present MRT-LB model is studied and compared with the LBGK model. The rest of this paper is organized as follows. The macroscopic governing equations are described in Section 2. The MRT-LB model and the boundary conditions are presented in Section 3. The numerical tests and some discussions are given in Section 4. Finally, a brief conclusion is made in Section 5.

2. Macroscopic governing equations

The flow is supposed to be two-dimensional, laminar and incompressible with negligible viscous heat dissipation. In addition, it is assumed that the solid phase is in local thermal equilibrium with the fluid phase [2]. Based on the generalized model, the dimensional governing equations for convection heat transfer in a homogeneous, isotropic and fluid-saturated porous medium at the REV scale can be written as [6,20,37,53]:

$$\nabla \cdot \mathbf{u} = 0, \tag{1a}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\left(\frac{\mathbf{u}}{\phi}\right) = -\frac{1}{\rho_0}\nabla(\phi p) + \upsilon_e \nabla^2 \mathbf{u} + \mathbf{F}, \tag{1b}$$

$$\sigma \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla \cdot (\alpha_e \nabla T) + Q''', \tag{1c}$$

where ρ_0 is the mean fluid density, $\mathbf{u}$, T, and p are the volume-averaged fluid velocity, temperature, and pressure, respectively; ϕ is the porosity, υ_e is the effective kinetic viscosity, and Q''' is the internal heat source term; the coefficient $\sigma = [\phi(\rho_f c_{pf}) + (1-\phi)(\rho_s c_{ps})]/(\rho_f c_{pf})$ is the thermal capacity ratio, in which ρ_f (ρ_s) and c_{pf} (c_{ps}) are the density and specific heat of the fluid (solid) phase, respectively; α_e is the effective thermal diffusivity $\alpha_e = k_e/(\rho_f c_{pf})$, in which k_e is the effective thermal conductivity of the porous medium. $\mathbf{F} = (F_x, F_y)$ denotes the total body force induced by the porous media and other external forces, which can be expressed as [37]

$$\mathbf{F} = -\frac{\phi \upsilon}{K}\mathbf{u} - \frac{\phi F_\phi}{\sqrt{K}}|\mathbf{u}|\mathbf{u} + \phi \mathbf{G}, \tag{2}$$

where K is the permeability, $|\mathbf{u}| = \sqrt{u_x^2 + u_y^2}$, in which u_x and u_y are the components of $\mathbf{u}$ in the x- and y-directions, respectively, υ is the kinetic viscosity of the fluid (υ is not necessarily the same as υ_e). According to the Boussinesq approximation, the body force $\mathbf{G}$ is given by

$$\mathbf{G} = g\beta(T - T_0)\mathbf{j} + \mathbf{a} \tag{3}$$

with the first term denoting the buoyancy force (g is the gravitational acceleration, β the thermal expansion coefficient, T_0 the reference temperature, $\mathbf{j}$ the unit vector in the y-direction), and the second term representing the acceleration induced by other external forces. Based on Ergun's experimental relation [38], the geometric function F_ϕ and the permeability K can be expressed as [39]

$$F_\phi = \frac{1.75}{\sqrt{150\phi^3}}, \quad K = \frac{\phi^3 d_p^2}{150(1-\phi)^2}, \tag{4}$$

where d_p is the solid particle diameter.

The system governed by Eq. (1) is characterized by several dimensionless parameters: the Darcy number Da, the Rayleigh number Ra, the internal Rayleigh number Ra_I (for thermal convection flows with internal heat source), the Reynolds number Re (for mixed convection flow), the Prandtl number Pr, the viscosity ratio J, which are defined as follows

$$Da = \frac{K}{L^2}, \quad Ra = \frac{g\beta\Delta T L^3}{\upsilon \alpha_e}, \quad Ra_I = \frac{g\beta Q''' L^5}{\upsilon \alpha_e^2}, \quad Re = \frac{LU}{\upsilon}, \quad Pr = \frac{\upsilon}{\alpha_e}, \quad J = \frac{\upsilon_e}{\upsilon}, \tag{5}$$

where L is the characteristic length, ΔT is the temperature difference (characteristic temperature), and U is the characteristic velocity.

3. MRT-LB model for thermal flows in porous media

3.1. D2Q9 MRT-LB equation for the flow field

For the flow field, a LB equation with the MRT collision model [23–27] is considered in this study. According to Refs. [28,30], the MRT-LB equation with an explicit treatment of the forcing term can be written as

$$\mathbf{f}(\mathbf{x}_k + \mathbf{e}\delta_t, t_n + \delta_t) - \mathbf{f}(\mathbf{x}_k, t_n) = -\mathbf{M}^{-1}\boldsymbol{\Lambda}[\mathbf{m} - \mathbf{m}^{(\mathrm{eq})}]|_{(\mathbf{x}_k, t_n)} + \mathbf{M}^{-1}\delta_t\left(\mathbf{I} - \frac{\boldsymbol{\Lambda}}{2}\right)\mathbf{S}, \tag{6}$$

where $\mathbf{M}$ is a $Q \times Q$ orthogonal transformation matrix (Q represents the number of discrete velocities), $\boldsymbol{\Lambda} = \mathbf{M}\tilde{\boldsymbol{\Lambda}}\mathbf{M}^{-1} = \mathrm{diag}(s_0, s_1, \ldots, s_{Q-1})$ is a diagonal relaxation matrix ($\tilde{\boldsymbol{\Lambda}}$ is the collision matrix and $\{s_i | 0 \leqslant i \leqslant Q-1\}$ are relaxation rates), and $\mathbf{I}$ is the unit matrix. The