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Analysis of the conduction–radiation problem in absorbing, emitting, non-gray planar media using an exact method

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ABSTRACT

Systems in which both conduction and radiation are the dominant modes of heat transfer are important in many engineering applications and various numerical methods exist to analyze such systems. An exact solution to the conduction–radiation problem in a one-dimensional, planar, absorbing, emitting, non-gray medium is presented. The method uses an integrating factor to solve the radiative transfer equation and variation of parameters is used to solve the energy equation. The model is verified by comparing the temperature profiles calculated from this work to those found using numerical methods for both gray and non-gray cases.

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1. Introduction

Systems in which both conduction and radiation are the dominant modes of heat transfer can be found in many practical engineering applications. These include fiber and foam insulations, the manufacture of glass, the study and design of furnaces and internal-combustion engines, filler and cover for windows and solar collectors, thermal barrier coatings, and many others. The wide variety of applications has resulted in several numerical and experimental studies of methods for analyzing systems with simultaneous conduction and radiation.

The analysis of such systems is inherently difficult because of the integro-differential nature of the radiative transfer equation (RTE) [1]. The pioneering theoretical analysis for this problem was presented by Viskanta and Grosh in 1962 in which the temperature profile of a one-dimensional, gray medium bounded on both sides by opaque surfaces was obtained by transforming the governing integro-differential equation into a non-linear integral equation that was solved iteratively [2]. The same authors investigated the effects of different emittances of the bounding surfaces on the heat transfer in the gray medium [3]. The results of these analyzes are often used as benchmark solutions to which other methods are compared.

More recent approaches to solving the combined conduction–radiation problem include the use of the finite element method [4,5], the finite difference method [6,7], the finite volume method [8,9], and the lattice Boltzmann method [10–12]. The radiation portion of the problem has been solved using the discrete ordinates method [11], the discrete transfer method [1,12], the method of spherical harmonics [1,13], the Monte Carlo method [14], and the finite volume method [10,15]. Each of these methods has advantages and disadvantages [16].

Most of the published solutions deal with heat transfer in gray media [17]. Fewer investigations have been conducted for the case of conduction–radiation heat transfer in non-gray media. Heinemann et al. used theoretical and experimental methods to analyze conduction–radiation heat transfer in non-gray silica aerogels [18] while Manohar et al. investigated conduction–radiation heat transfer in non-gray plastics [19]. More recently, Marques et al. developed a computationally efficient numerical model based on finite strip theory to determine the temperature profile and heat flux in absorbing, emitting, non-scattering, non-gray media [17].

The solution presented here resulted from an investigation into the temperature gradient in thermal barrier coatings (TBCs). Thermal barrier coatings allow for increased inlet temperatures in power generation and aerospace turbines, thereby increasing efficiency and reducing air cooling requirements. Knowledge of the temperature profile in a thermal barrier coating is critical for evaluating the TBC performance and monitoring its health, as well as for accurate simulation and modeling. Another current application of the solution presented in this paper is determination of the

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Nomenclature

E	emissive power
H	terms in summation of temperature profile
I	radiative intensity
k	thermal conductivity
L	length of medium
N	conduction-to-radiation parameter
P	function of z, λ, and μ
Q	function of z, λ, and μ
q	heat flux
R	function of λ and μ
$\hat{s}$	direction vector
T	temperature
z	Cartesian coordinate
Greek symbols	
ε	emittance of boundary
θ	polar angle
κ	absorption coefficient
λ	wavelength
μ	cosine of intensity direction
ρ	reflectance of boundary
σ	Stefan–Boltzmann constant
ϕ	azimuthal angle
Ω	solid angle
Subscripts	
0	at left boundary
b	blackbody
L	at right boundary
i, m, n	indices
R	radiative
s	in the $\hat{s}$ direction
λ	spectrally dependent
Superscripts	
+	forward direction
−	backward direction
′	integration coordinate

temperature profile in blackbody optical fiber pyrometers [20–22]. A blackbody optical fiber pyrometer consists of an optical fiber whose sensing tip is coated with a highly conductive, opaque material. The blackbody radiation emitted by the tip of the fiber is transmitted along the optical fiber to a detector. Correct interpretation of these emission measurements requires a solution to the coupled radiation–conduction problem. The use of blackbody optical fiber pyrometers in oxy-combustion flames is currently being investigated. Oxy-combustion is a promising technology being considered for carbon capture and sequestration due to the fact it has the potential to produce very low levels of emissions for all of the major pollutants of coal and natural gas. An understanding of the gas temperatures is critical to oxy-combustion development. Blackbody optical fiber pyrometry has the potential to solve all of the major problems associated with measuring temperatures in large-scale, particle-laden combustion systems that are required for the development of oxy-combustion.

Previous efforts to solve combined conduction and radiation heat transfer problems have relied on numerical methods to solve the governing equations. The present work outlines an exact solution to the equations governing the simultaneous conduction and radiation heat transfer in a one-dimensional, plane parallel, absorbing, emitting, non-scattering, non-gray medium surrounded by diffuse, opaque surfaces. The particular solution to the governing differential equation is obtained using the method of variation of parameters while the spectral intensities required to calculate the total radiative heat flux are found by solving the RTE using an integrating factor. This approach results in an integral equation that is solved for the temperature profile. The temperature profile is obtained using iterative, numerical integration, and a closed-form solution is not obtained. However, since numerical integration can be performed to an arbitrary degree of precision, the solution is exact. The model is verified by comparing the results for various cases to those calculated using different numerical methods and to CFD simulations performed using commercial software [23], which employs the discrete ordinates method to model the radiative heat transfer.

2. Problem formulation

A one-dimensional, plane-parallel, homogeneous, isotropic, non-gray, participating medium bounded by two surfaces is shown in Fig. 1. The medium is absorbing, emitting, and non-scattering and the bounding surfaces are opaque and diffuse. The temperature of the boundaries are denoted by T_0 and T_L at $z = 0$ and $z = L$, respectively. The material properties of both the participating medium and the boundaries are independent of temperature. Heat transfer in the medium occurs by both conduction and radiation. The energy equation for simultaneous conduction and radiation for a one-dimensional, planar medium at steady state with uniform properties reduces to [1]

$$\frac{d^2T}{dz^2} = \frac{1}{k}\frac{dq_R}{dz} \tag{1}$$

Eq. (1) is subject to the following boundary conditions.

$$T(0) = T_0 \tag{2}$$

$$T(L) = T_L \tag{3}$$

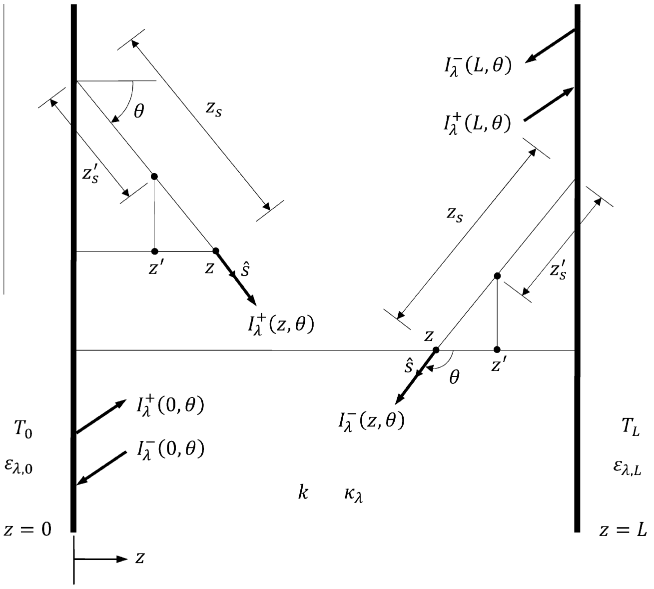

Fig. 1. Absorbing–emitting medium between two diffuse boundaries. Coordinates used in the theoretical analysis are shown.

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