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Mixed convection stagnation flow towards a vertical shrinking sheet

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ABSTRACT

The steady mixed convection stagnation-point boundary layer flow past a vertical stretching/shrinking sheet in a viscous fluid is numerically investigated. The numerical solutions are obtained by solving the similarity equations which are derived via the similarity transformation technique in order to reduce the nonlinear partial differential equations into a system of nonlinear ordinary differential equations. Numerical techniques, namely the Keller-box method, along with the shooting method are used to solve the transformed ordinary differential equations. Results have been presented and discussed for the effects of the governing parameters on the skin friction coefficient and the local Nusselt number as well as for the velocity and temperature profiles. The present results show quite interesting flow behavior for the shrinking sheet problem compared to the stretching sheet problem. On the other hand, it is also found that dual solutions exist for the present problem. The streamlines for the two dimensional flow have also been presented.

1. Introduction

A stagnation point occurs whenever a flow impinges on a solid object and it holds the highest pressure and the highest heat transfer. Hiemenz [1] was the first who investigated the two dimensional stagnation flow towards a stationary semi-infinite wall by using similarity transformation in order to reduce Navier–Stokes equation to a nonlinear ordinary differential equation. The exact similar solutions for the thermal field were later reported by Eckert [2]. An analysis of mixed convection near the stagnation flow of a vertical surface has been carried out by Ramachandran et al. [3] for an arbitrary variation of the surface temperature or the surface heat flux conditions. This paper has been later extended by Lok et al. [4] to the case of unsteady micropolar fluids. On the other hand, mixed convection is the combination between forced and natural or free convection flows. Both free and mixed convection flows are due to the buoyancy effects. This type of convection happens due to the temperature difference between the wall and the free stream and thus changes the fluid flow and heat transfer. During the past few decades, the flows of free and mixed convection have been analyzed and studied by many investigators. This is because of their importance in many applications in industrial and manufacturing processes, such as, welding, extrusion of plastics, paper drying, hot rolling, etc. Related to the mixed convection topic, we mention the papers by Seshadri et al. [5], Ishak et al. [6], Magyari and Aly [7] and many more for different situations. Nazar et al. [8] considered the unsteady mixed convection boundary layer flow near the stagnation point on a vertical surface in a porous medium. Several researchers also investigated the problem of mixed convection flow past a vertical surface and also the stagnation point flow for a non-Newtonian fluid. Abbas et al. [9] considered a steady mixed convection boundary layer flow of an incompressible Maxwell fluid near the two dimensional stagnation point flow over a vertical stretching surface. Hayat et al. [10] considered this problem for a viscoelastic fluid flow.

Wang [11] studied the stagnation point flow towards a shrinking sheet by considering two dimensional and axisymmetric cases. He showed that the non-alignment of the stagnation flow and the shrinking sheet complicates the flow structure. Moreover, he found that solutions do not exist for large shrinking rates and may be non-unique in the two-dimensional case. Kimiaefar et al. [12] proposed an analytical approach to study the case of stagnation point flow in the vicinity of a shrinking sheet by using Homotopy Analysis Method (HAM). Recently, Ishak et al. [13] studied a problem of two dimensional stagnation point flow of an incompressible fluid over a stretching vertical sheet in its own plane. Further, Bachok et al. [14] examined the steady two dimensional stagnation point flow and heat transfer from a warm, laminar liquid flow to a melting stretching/shrinking sheet. Different from the stretching

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Nomenclature

a, b, c, d constants
A constant for the boundary layer displacement
g acceleration due to gravity
Gr_x local Grashof number
l characteristic length of the flat plate
p dimensionless pressure
$\bar{p}$ dimensional pressure
p_e dimensionless pressure far from the surface of the sheet (inviscid flow)
$\bar{p}_e$ dimensional pressure far from the surface of the sheet (inviscid flow)
Pr Prandtl number
q_w heat transfer from the shrinking surface
Re_x local Reynolds number
T dimensionless fluid temperature
$\overline{T}$ dimensional fluid temperature
T_w dimensionless temperature of the shrinking sheet
$\overline{T}_w$ dimensional temperature of the shrinking sheet along $\bar{x}$ direction
T_∞ ambient temperature
u, v dimensionless velocity components along the x and y directions, respectively
$\bar{u}, \bar{v}$ dimensional velocity components along the $\bar{x}$ and $\bar{y}$ directions, respectively
u_e dimensionless velocity of external flow along the x direction
$\bar{u}_e$ dimensional velocity of external flow along the $\bar{x}$ direction
u_w dimensionless velocity of the shrinking surface along the x direction
$\bar{u}_w$ dimensional velocity of the shrinking surface along the $\bar{x}$ direction
v_e dimensionless velocity of external flow along the y direction
$\bar{v}_e$ dimensional velocity of external flow along the $\bar{y}$ direction
x, y dimensionless Cartesian coordinates along the surface and normal to it, respectively
$\bar{x}, \bar{y}$ dimensional Cartesian coordinates along the surface and normal to it, respectively

Greek symbols
α thermal diffusivity
β thermal expansion coefficient
δ non-alignment distance
γ constant velocity ratio parameter
η pseudo-similarity variable
λ constant mixed convection parameter
ν kinematic viscosity
θ dimensionless temperature
ρ fluid density
τ_w skin friction from the surface of the sheet

Superscript
$'$ differentiation with respect to η

Subscripts
w condition at the shrinking sheet
∞ condition at infinity

Table 1
Comparison of results for the values of $F''(0)$ and $G''(0)$ for several values of γ when δ = 1, Pr = 0.78 and $\lambda = 0$ (forced convection).

λ	γ	Kimiaeifar et al. [12]		Bachok et al. [14]	Wang [11]		Present result	
		$F''(0)$	$G''(0)$	$F''(0)$	$F''(0)$	$G''(0)$	$F''(0)$	$G''(0)$
0	0	1.23259	−0.81130	1.23259	1.23259	−0.81130	1.23259	−0.81130
	0.1	1.14656	−0.86345	1.14656	1.14656	−0.86345	1.14656	−0.86345
	0.2	1.05114	−0.91331	1.05113	1.05113	−0.91330	1.05113	−0.91330
	0.5	0.71330	−1.05146	0.71329	0.71330	−1.05239	0.71329	−1.05146
	1	0.00000	−1.25332	–	0.00000	−1.25331	0.00000	−1.25331
	2	−1.88735	−1.58957	−1.88731	−1.88731	−1.58957	−1.88731	−1.58957
	5	−10.2651	−2.33810	–	−10.26475	−2.33810	−10.26475	−2.33810
	−0.25	1.40225	−0.66857	1.40224	1.40224	−0.66857	1.40224	−0.66857
	−0.5	1.49567	−0.50145	1.49567	1.49567	−0.50145	1.49567	−0.50145
	−0.75	1.48934	−0.29376	1.48930	1.48930	−0.29376	1.48930	−0.29377
	−0.999	1.32888	0.00000	–	–	–	1.32993	−0.00150
	−1	–	–	1.32882	1.32882	0.00000	1.32882	0.00000
	−1.1	–	–	1.18668	–	–	1.18668	0.17696
	−1.15	–	–	1.08223	1.08223	0.29800	1.08223	0.29800
	−1.2	–	–	0.93247	–	–	0.93247	0.47186
	−1.2465	–	–	0.58430	0.55430	0.99904	0.58428	0.94777
	−1.24657	–	–	0.56397	–	–	0.56401	0.98214

sheet, they found that the solutions for a shrinking sheet are non-unique. It is worth mentioning to this end that important and new results on the flow induced by a shrinking sheet in a viscous fluid were recently presented by Fan et al. [15], Bhattacharyya [16] and several other authors. Very recently, Rohni et al. [17] investigated theoretically the problem of steady laminar two-dimensional boundary layer flow and heat transfer of an incompressible viscous fluid at a stagnation point over an exponentially shrinking vertical sheet with suction.

Inspired by the study done by Wang [11] and Ishak et al. [13], the present paper deals with the steady stagnation flow towards a vertical shrinking sheet. It should be noticed that the stretching/shrinking axis and the stagnation flow are, in general, not aligned. All the previous authors considered only the aligned cases of the present problem. To the best of our knowledge, this problem has not been studied before and the results reported here are new and original.

2. Basic equations

Consider the steady mixed convection flow close to a non-alignment stagnation-point and a vertical shrinking surface in a viscous fluid. Following Wang [11], it is assumed that the velocity of the inviscid (potential) fluid is $\bar{u}_e(\bar{x}) = a\bar{x}$ and that of the stretching/shrinking sheet is $\bar{u}_w(x) = b(\bar{x} + c)$, where a is a positive constant,