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The asymptotic solutions of heat problem of friction for a three-element tribosystem with generalized boundary conditions on the surface of sliding

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ABSTRACT

The asymptotic (at large and small values of the dimensionless time) solutions are obtained to the heat problem of friction for the three-element tribosystem – the semi-space/the strip/the semi-infinity foundation. Comparison of values of the temperature obtained by means of exact (in the quadratures or the functional series) and asymptotic solutions at different values of the input parameters is executed. This allows to establish of the time limits their applications.

1. Introduction

The review of the current state of mathematical modeling of frictional heating by means of solutions to one-dimensional thermal problems of friction has been presented in the article [1]. The detailed formulations and exact solutions of such problems for a three-element tribosystem, consisting of a semi-space, sliding along a surface of a strip deposited on a semi-infinite foundation, have been studied in the articles [2–6]. The conditions of perfect [7] or imperfect [8,9] thermal contact during friction of bodies were used in the formulation of these problems. Subsequently, the analytical (in quadratures) solution of the thermal problem of friction for three-element tribosystem with generalized Barber's boundary conditions [10,11] on the sliding surface has been obtained in the article [12]. In the specific case, when the materials of a strip and a foundation are the same, this solution has been integrated, and the exact and asymptotic (for small and large values of the dimensionless time) solutions for the two semi-spaces have been found.

The objective of this article is to obtain the asymptotic solutions for a three-element tribosystem. We note that obtaining a sufficient simple formulae to find the temperature of the various tribosystems on the basis of asymptotic solutions is important from the point of view of engineering applications. Requiring no knowledge, neither a complex mathematical apparatus of integral transforms nor numerical integration, the asymptotic solutions allow to quickly and accurately estimate the temperature state of the friction couple.

2. Statement of the problem

Consider a system consisting of a top and a lower semi-spaces (Fig. 1). The top semi-space is homogeneous and the lower is a piecewise-homogeneous. The last represents a strip of thickness d placed on a semi-infinite foundation. In initial time moment $t = 0$ the bodies are compressed by a constant normal pressure p_0 acting at infinity parallel to the z-axis of the Cartesian coordinate system $Oxyz$. The top semi-space slides with the constant speed V in the direction of the y-axis on the surface of the strip. Due to the friction on the contact surface $z = 0$ the heat generation occurs, that leads to the heating of the entire system. It is assumed that:

(1) the thermophysical properties of the bodies are independent of temperature;
(2) the sum of the intensities of heat fluxes on the contact surface $z = 0$ directed along the normal to this surface into each the semi-spaces is equal to the specific power of friction $q_0 = fVp_0$;
(3) due to the thermal resistance, there is a heat transfer with a constant coefficient of thermal conductivity of the contact h through the contact surface of the top semi-space and the strip;

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Nomenclature

$\mathrm{Bi} = hd/K_s$	Biot number
d	thickness of the strip
$\mathrm{erf}(x)$	Gauss error function
$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$	complementary error function
$\mathrm{ierfc}(x)\pi^{-1/2}\exp(-x^2) - x\,\mathrm{erfc}(x)$	integral of the error function $\mathrm{erfc}(x)$
f	coefficient of friction
h	coefficient of thermal conductivity of contact
K	coefficient of heat conduction
k	coefficient of thermal diffusivity
p_0	pressure
$q_0 = fVp_0$	intensity of the frictional heat flux (the power of friction)
T	temperature
$T_0 = \frac{q_0 d}{K_s}$	temperature scaling factor
$T^* = T/T_0$	dimensionless temperature
t	time
V	sliding speed
z	spatial coordinate

Greek symbols

$\tau = k_s t/d^2$	dimensionless time (Fourier's number)
$\zeta = z/d$	dimensionless coordinate

Subscripts

f	foundation
s	strip
t	top semi-space

(4) the temperatures and heat fluxes of the strip and the foundation are the same on the interface $z = -d$;
(5) the wear of the rubbing surfaces is neglected.

Further, all values and parameters concerning the top semi-space, strip and foundation will have bottom indexes "t", "s", and "f", respectively.

On the above-mentioned assumptions, the corresponding heat problem of friction for the considered three-element tribosystem takes the form:

$$\frac{\partial^2 T_t^*(\zeta,\tau)}{\partial\zeta^2} = \frac{1}{k_t^*}\frac{\partial T_t^*(\zeta,\tau)}{\partial\tau}, \quad \zeta > 0,\ \tau > 0, \tag{1}$$

$$\frac{\partial^2 T_s^*(\zeta,\tau)}{\partial\zeta^2} = \frac{\partial T_s^*(\zeta,\tau)}{\partial\tau}, \quad -1 < \zeta < 0, \quad \tau > 0, \tag{2}$$

$$\frac{\partial^2 T_f^*(\zeta,\tau)}{\partial\zeta^2} = \frac{1}{k_f^*}\frac{\partial T_f^*(\zeta,\tau)}{\partial\tau}, \quad \zeta < -1, \quad \tau > 0, \tag{3}$$

$$-K_t^*\frac{\partial T_t^*(\zeta,\tau)}{\partial\zeta}\bigg|_{\zeta=0} = \gamma - \mathrm{Bi}\left[T_t^*(0,\tau) - T_s^*(0,\tau)\right], \quad \tau > 0, \tag{4}$$

$$\frac{\partial T_s^*(\zeta,\tau)}{\partial\zeta}\bigg|_{\zeta=0} = 1 - \gamma + \mathrm{Bi}\left[T_t^*(0,\tau) - T_s^*(0,\tau)\right], \quad \tau > 0, \tag{5}$$

$$T_s^*(-1,\tau) = T_f^*(-1,\tau), \quad \tau > 0, \tag{6}$$

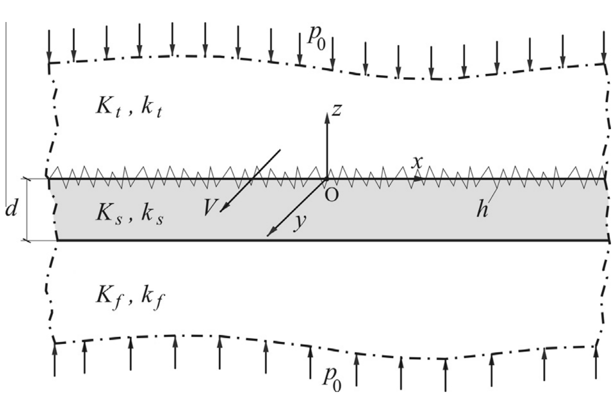

Fig. 1. Scheme of the three-element sliding system.

$$\frac{\partial T_s^*}{\partial\zeta}\bigg|_{\zeta=-1} = K_f^*\frac{\partial T_f^*}{\partial\zeta}\bigg|_{\zeta=-1}, \quad \tau > 0, \tag{7}$$

$$T_{t,f}^*(\zeta,\tau) \to 0, \quad |\zeta| \to \infty, \quad \tau > 0, \tag{8}$$

$$T_{t,s,f}^*(\zeta,0) = 0, \quad |\zeta| < \infty, \tag{9}$$

where

$$\zeta = \frac{z}{d},\ \tau = \frac{k_s t}{d^2},\ K_{t,f}^* = \frac{K_{t,f}}{K_s},\ k_{t,f}^* = \frac{k_{t,f}}{k_s},\ \mathrm{Bi} = \frac{hd}{K_s},\ \ T_{t,s,f}^* = \frac{T_{t,s,f}}{T_0},\ \ T_0 = \frac{q_0 d}{K_s}. \tag{10}$$

3. Exact solution of the problem

Applying to the boundary-value heat conduction problem (1)–(9) the Laplace integral transform [13]

$$\overline{T}_{t,s,f}^*(\zeta,p) \equiv L\left[T_{t,s,f}^*(\zeta,\tau);p\right] = \int_0^\infty T_{t,s,f}^*(\zeta,\tau)e^{-p\tau}d\tau, \quad \tau \geq 0, \tag{11}$$

we find:

$$\overline{T}_{t,s,f}^*(\zeta,p) = \frac{\Delta_{t,s,f}(\zeta,p)}{p\sqrt{p}\,\overline{\Delta}(p)}, \tag{12}$$

$$\Delta_t(\zeta,p) = \left[(\varepsilon_f \mathrm{Bi} + \gamma\sqrt{p})\mathrm{sh}\sqrt{p} + (\mathrm{Bi} + \gamma\varepsilon_f\sqrt{p})\mathrm{ch}\sqrt{p}\right]e^{-\zeta_t\sqrt{p}}, \quad \zeta \geq 0, \tag{13}$$

$$\Delta s(\zeta,p) = [\mathrm{Bi} + (1-\gamma)\varepsilon_t\sqrt{p}]\{\varepsilon_f \mathrm{sh}[(1+\zeta)\sqrt{p}] + \mathrm{ch}[(1+\zeta)\sqrt{p}]\}, \quad -1 \leq \zeta \leq 0, \tag{14}$$

$$\Delta_f(\zeta,p) = [\mathrm{Bi} + (1-\gamma)\varepsilon_t\sqrt{p}]e^{(1-\zeta_f)\sqrt{p}}, \quad \zeta \leq -1, \tag{15}$$

$$\overline{\Delta}(p) = [(1+\varepsilon_t\varepsilon_f)\mathrm{Bi} + \varepsilon_t\sqrt{p}]\mathrm{sh}\sqrt{p} + [(\varepsilon_t + \varepsilon_f)\mathrm{Bi} + \varepsilon_t\varepsilon_f\sqrt{p}]\mathrm{ch}\sqrt{p}, \tag{16}$$

where

$$\zeta_t = \frac{\zeta}{\sqrt{k_t^*}},\ \zeta_f = 1 - \frac{(1+\zeta)}{\sqrt{k_f^*}},\ \varepsilon_{t,f} = \frac{K_{t,f}^*}{\sqrt{k_{t,f}^*}}. \tag{17}$$

The inverse of the transform solution (12)–(16), that satisfies the boundary conditions (1)–(9), has the form [12]:

$$T_{t,s,f}^*(\zeta,\tau) = \frac{2}{\pi}\int_0^\infty \frac{G_{t,s,f}(\zeta,\tau)}{\Delta(x)}P(\tau,x)dx, \ \tau \geq 0, \tag{18}$$

$$G_t(\zeta,x) = [a(x)\Delta_R(x) + b(x)\Delta_I(x)]\cos(\zeta_t x) + [b(x)\Delta_R(x) - a(x)\Delta_I(x)]\sin(\zeta_t x), \quad \zeta \geq 0, \tag{19}$$