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Glass sagging simulation with improved calculation of radiative heat transfer by the optimized reciprocity Monte Carlo method

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ABSTRACT

Glass sagging is used to process glass industrial products such as windscreens, mirrors or lenses. A 2D glass sagging process, simulated with the Finite Element Method (FEM), is presented in this work. Different thermal cases are reviewed with special care brought to radiative transfer model, with an optimized reciprocity Monte Carlo method used as the reference. Results show that ignoring radiative transfer is a too rough hypothesis. This leads to large errors on the glass temperature distribution, on the forming process and on the final shape in case of glass sagging without mold. However, predefining glass temperature or using Rosseland approximation give acceptable results, less accurate than Monte Carlo simulations especially for a fine prediction of the transfer as a function of time, but with smaller CPU times.

1. Introduction

Glass sagging operation consists in forming a sheet or a plate of glass by heating it in a furnace. Glass temperature rises and reaches a work temperature at which viscosity is low enough to allow glass to sag under its own weight due to gravity.

In the case of windscreen processing, glass sagging is the most often used forming process [1,2]. Automotive windscreen production requires glass sheet bending in order to obtain complex shapes and a good optical surface. Glass sheets are placed on a ring frame and the forming phase ends when windscreen reaches the desired shape [3]. Glass sagging is also used to produce spherical, aspherical and even freeform mirrors [4] or lenses [5]. These mirrors can be used in many industrial applications such as solar panels or scientific instruments, e.g., astronomical telescopes [6]. For these products, the process ends when the object totally fits the shape of a mold placed below as shown in Fig. 1.

In theory, glass sagging looks like a simple operation process, but in practice, parameters need to be tuned accurately. If not, some flaws can happen, for example:

- sticking problems between glass and mold [7],
- heterogeneity of plate thickness [8],
- optical surface roughness [9],
- important residual mechanical stress [10,11].

These problems lead to material waste and expensive production cost.

The main complexity of the process lies on the thermomechanical coupling. Glass viscosity is very dependent on temperature: at the working stage, a temperature variation of a few dozens degrees can change the viscosity by several decades [12]. Conversely, when glass shape changes, it modifies the thermal flux inside. Some authors conducted glass sagging experiments in order to better understand and improve the process [4]. Experimental results can also be used to validate numerical modeling of the forming operation [2,8]. Computer simulations are generally less expensive and time consuming than experiments. They can also give many information on the influence of process parameters on the final product [13].

Numerically, mechanical equations are usually solved using a Finite Element Method (FEM) as in [14,15]. Concerning the thermal part, different methods have been used to determine temperature fields during glass forming processes. Two main hypotheses are usually found in the literature: glass temperature is directly set without calculations [16] or temperature is evaluated thanks to a heat transfer model but neglecting radiative transfer [17]. However, this last hypothesis could be too rough to give accurate temperature distributions [18].

In this work, a two-dimensional glass sagging case is modeled and different hypotheses on temperature distribution are

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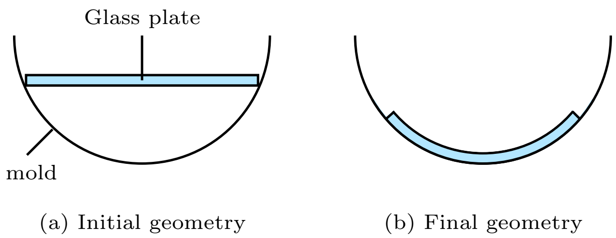

Fig. 1. Glass sagging process.

reviewed. The setup involves a glass plate processed on a concave mold. Mechanical modeling is achieved by FEM and glass behavior is assumed to obey a Maxwell law at working temperature.

Concerning the thermal part of the problem, four cases are considered:

(i) Glass temperature is set uniform inside the plate;
(ii) During the sagging process, conduction inside the medium and convection at the boundaries are considered but radiative transfer is neglected;
(iii) Radiation is taken into account in addition to the other heat transfer modes using Rosseland approximation;
(iv) Radiation calculations are conducted with an optimized reciprocity Monte Carlo method (ORM).

The last case (iv) is considered as the reference in this present work as Monte Carlo methods are known to give accurate results, in spite of heavier computational (CPU) times [19]. Comparison of cases (i)–(ii) with case (iv) will highlight the relevance of taking radiative transfer into account during the simulation of the glass sagging process. Then, comparing (iii) and (iv) simulation results will give interesting information on the accuracy of Rosseland approximation, knowing that the concept of radiative conductivity derived from Rosseland model is sometimes used for glass forming process simulation [20].

The first part of the present paper describes the physical model of sagging process. Then, the second part deals with thermal numerical schemes and the validation of the Monte Carlo code. The next section is dedicated to the numerical setup and finally simulation results are presented and discussed.

2. Physical model for sagging process

Abaqus® is used in the present work in order to address the thermomechanical problem by the FEM. The model presentation is only focused on the main parameters involved in the mechanical behavior laws and the specific model of radiative transfer which has been developed and combined to the FEM code.

2.1. Mechanical behavior laws

The continuity equation gives the displacement vector $\mathbf{u}$ as a function of the stress tensor $\boldsymbol{\sigma}$:

$$\rho\ddot{\mathbf{u}} = \mathbf{div}\,\boldsymbol{\sigma} + \mathbf{f}, \tag{1}$$

where ρ stands for the density and $\mathbf{f}$ is the body forces vector. Then, in case of small deformation, displacements are linked with the strain tensor $\boldsymbol{\epsilon}$ according to the relation:

$$\boldsymbol{\epsilon} = \frac{1}{2}(\mathbf{grad}(\mathbf{u}) + {}^{t}\mathbf{grad}(\mathbf{u})). \tag{2}$$

Making the assumption that glass behaves like a Maxwell rheological material [21] (from a rheological point of view, it corresponds to the connection in series of a dashpot behaving like a Newtonian fluid with a spring obeying a Hooke's law) which undergoes thermal expansion, strain tensor $\boldsymbol{\epsilon}$ can be written as the sum of elastic, viscous and thermal strains:

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^v + \boldsymbol{\epsilon}^t. \tag{3}$$

$\boldsymbol{\epsilon}^e$ is the elastic strain tensor:

$$\boldsymbol{\epsilon}^e = \frac{1}{2G}\boldsymbol{\sigma} + \frac{1}{3}\left(\frac{1}{3B} - \frac{1}{2G}\right)\mathrm{tr}(\boldsymbol{\sigma})\boldsymbol{I}. \tag{4}$$

tr is the matrix trace and $\boldsymbol{I}$ is the identity matrix. G is the shear modulus and B is the bulk modulus. They are expressed as a function of the Young's Modulus E and the Poisson's ratio ν_p:

$$G = \frac{E}{2(1+\nu_p)}, \tag{5}$$

$$B = \frac{E}{3(1-2\nu_p)}. \tag{6}$$

The viscous strain tensor $\boldsymbol{\epsilon}^v$ is linked to the stress tensor $\boldsymbol{\sigma}$ using the relation:

$$\dot{\boldsymbol{\epsilon}^v} = \frac{1}{2\eta_s}\boldsymbol{\sigma} + \frac{1}{3}\left(\frac{1}{3\eta_b} - \frac{1}{2\eta_s}\right)\mathrm{tr}(\boldsymbol{\sigma})\boldsymbol{I}. \tag{7}$$

By analogy with the elastic part, η_s and η_b can be assimilated to viscous shear modulus and viscous bulk modulus, respectively:

$$\eta_s = \frac{\eta}{2(1+\nu_\eta)}, \tag{8}$$

$$\eta_b = \frac{\eta}{3(1-2\nu_\eta)}. \tag{9}$$

ν_η is the viscous Poisson's ratio and is assumed to be equal to the Poisson's ratio. Viscosity η is the key parameter during glass sagging process. Its value is directly linked to the glass speed forming and is supposed to be controlled by a Williams–Landel–Ferry (WLF) law such as:

$$\lg\left(\frac{\eta(T)}{\eta(T_0)}\right) = \frac{-C_1(T-T_0)}{C_2+(T-T_0)}, \tag{10}$$

where $\eta(T_0)$, C_1 and C_2 are empirical coefficients.

Finally, $\boldsymbol{\epsilon}^t$ is the thermal expansion term:

$$\boldsymbol{\epsilon}^t = \frac{\alpha}{3}\Delta T\boldsymbol{I}. \tag{11}$$

α is the thermal expansion coefficient and ΔT is the temperature variation.

2.2. Heat balance equation and boundary conditions

In its classical form, the heat balance equation can be written:

$$\rho c_p \dot{T} = -\mathrm{div}(\mathbf{q_c} + \mathbf{q_r}), \tag{12}$$

where $\dot{T}$ is the temperature time derivative, c_p is the heat capacity per mass unit at constant pressure, $\mathbf{q_c}$ and $\mathbf{q_r}$ are the conductive and radiative heat flux vectors, respectively. Convection is neglected inside glass as deformation is very slow and piece deformation has a very little influence on temperature distribution. At the glass/air interface, boundary conditions can be written:

$$K(T_a - T_g) + \mathbf{q_r^{a\to g}}\cdot\mathbf{n} = \left[\mathbf{q_r^{g\to a}} + k_c\,\mathbf{grad}(T)\right]\cdot\mathbf{n}, \tag{13}$$

where $\mathbf{n}$ is the normal vector oriented outward from glass. T_a is the air temperature and T_g is the glass temperatures. K is a global heat exchange coefficient [22] and k_c is the thermal conductivity. $\mathbf{q_r^{i\to j}}$ corresponds to the radiative flux density crossing the interface between two media i and j. Radiative boundary conditions depend on the glass properties, namely in the semitransparent and the opaque ranges. $\mathbf{q_r^{i\to j}}$ can be written: