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# Analytical model of radiative properties of packed beds and dispersed media

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ABSTRACT

This paper presents an analytical formulation of radiative properties of statistically isotropic and homogeneous dispersed media in the geometric optic regime. It attempts to overcome the powerful but time consuming approaches based on ray-tracing Monte Carlo technique and advance the existing analytical models. We show that simple analytical formulas, allow to capture the main extinction mechanisms in a dispersed medium. The suitability of the proposed model is discussed through two comparative studies: firstly, with the ray-tracing Monte Carlo approach on radiative properties of packed beds of spheres and matrixes enclosing spherical cavities; and secondly, with the direct Monte Carlo simulation and literature data on hemispherical transmittances of packed beds of opaque and semi-transparent spheres. The computation time of the analytical approach is shown to be less significant than those of the ray-tracing Monte Carlo method.



## 1. Introduction

Dispersed media including packed and fluidized beds, composites, and porous materials present special features in term of thermal properties and specific area. They are thus very interesting materials for use in many engineering applications such as thermal insulation, biomedicine, solar or chemical to thermal energy conversion, and heat exchangers. Dispersed media absorb, emit and scatter the thermal radiation. The most common assumption to solve radiative transfer problem in such complex materials has been to treat the dispersed systems as continuous and homogeneous media characterized by the so-called "radiative properties" and then to use the radiative transfer equation (RTE) [1–3].

In the framework of the continuum radiative transfer theory, the thermal radiation propagation within an absorbing, emitting and scattering medium has been modeled by the RTE. Beyond the computational challenges in solving the RTE, a good knowledge of radiative properties, namely the absorption and scattering coefficients, and the scattering phase function, is of crucial importance.

The radiative properties of particulate media have been the subject of long term investigation. The radiative properties of a single typical particle (sphere or long cylinder) and a dilute mixture of them were reported in the majority of standard radiative transfer textbooks, such as in [1–3]. A review of experimental and theoretical determination of radiative properties of various dispersed media was reported by Baillis-Doermann and Sacadura [4], Sacadura and Baillis [5], and Sacadura [6]. The recent textbook by Dombrovsky and Baillis [7] reported engineering approaches and experimental strategies for determining spectral radiative properties of cellular foams, fibrous materials, porous ceramics, polymer based composites, and silica aerogel. Radiative properties of non-homogeneous and/or anisotropic dispersed materials were recently reported by Taine and Co-workers [8–10].

Focusing only on the theoretical aspect, the main prediction approaches can be classified in two categories depending on values of two characteristics ratios: (i) the ratio of the clearance between particles to the wavelength, namely $c/\lambda$; and (ii) the ratio of the clearance between particles to the particle size, namely $c/a$. The first category concerns dilute media characterized by large clearance to wavelength ratio ($c/\lambda \gg 1$) and large clearance to particle size ratio ($c/a \gg 1$). The criterion $c/\lambda \gg 1$ ensures that the radiation incident on each particle is a plane wave and each particle is located in the far-field zone of radiation sources [11]. The criterion $c/a \gg 1$ ensures that the scattered waves by close particles do not alter the radiation incident on each particle. It has been said that a "single scattering" occurs in each element volume [2]. In

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**Nomenclature**

| | |
|---|---|
| $a$ | sphere radius |
| $c$ | clearance between particles |
| $d$ | scatterer characteristic size |
| $D$ | parameter involved in Eqs. (6) and (20) |
| $E$ | slab thickness |
| $f_v$ | particle volume fraction |
| $F_{dep}$ | dependent scattering correction factor |
| $G$ | parameter defined by Eq. (6) |
| $I$ | radiation intensity |
| $I_b$ | blackbody radiation intensity |
| $k$ | imaginary part of complex refractive index |
| $l$ | distance or traveled path of a ray bundle |
| $L$ | geometric mean-beam-length |
| $m$ | complex refractive index |
| $m_p$ | number of particles |
| $n$ | real part of complex refractive index |
| $N$ | ray number |
| $N_{dir}$ | number of scattering angles |
| $Q$ | efficiency factor |
| $S$ | surface |
| $T_h$ | hemispherical transmittance |
| $V$ | volume |
| $x$ | size parameter |
| $z$ | axial coordinate |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | absorption coefficient |
| $\beta$ | extinction coefficient |
| $\chi$ | refraction angle |
| $\delta$ | parameter in Eq. (28) |
| $\phi$ | incidence angle |
| $\vartheta$ | parameter in Eq. (28) |
| $\dot{\eta}$ | number density of particles having radius between $a$ and $a + da$ |
| $\varphi$ | azimuth angle |
| $\lambda$ | wavelength |
| $\mu$ | direction cosine |
| $\Theta$ | scattering angle |
| $\rho$ | reflectivity |
| $\sigma$ | scattering coefficient |
| $\omega$ | scattering albedo |
| $\Omega$ | solid angle |
| $\xi$ | random number |
| $< >$ | averaging over particle size distribution |

*Subscripts and superscripts*

| | |
|---|---|
| abs | absorption |
| back | rays scattered from the illuminated area of particles |
| ext | extinction |
| h | hemispherical reflectivity |
| ind | independent scattering |
| $k_0$=0 | non-participating host medium |
| max | maximal ray path inside a particle |
| sca | scattering |
| trans | ray-transportation, transmission efficiency factor |
| // | parallel polarization |
| $\perp$ | perpendicular polarization |
| 0 | host medium |
| 1 | scatterers |

addition, closest particles do not obstruct the radiation incident on each particle. In other words, there is no "shadowing effects". In the presence of single scattering and negligible shadowing effect, the particles "scatter independently" the radiation [1–3,12]. Therefore, the scattering and absorption characteristics of the medium can be calculated from a linear summation rule, known as "independent scattering theory" [12–15] as discussed in Section 2.1.4. Soot, fluidized beds, silica aerogel, glass foams, porous ceramics, and polymer coatings pigmented with glass or metal particles have been often treated in the framework of independent scattering theory [16–20]. The independent scattering theory has been also combined with normal–normal or normal–hemispherical spectrometric measurements for improving the spectral radiative properties determination [21–24].

The second category of prediction methods concerns dispersed media consisting of closely spaced particles. With such a type of media, the radiative transfer becomes more complex. In fact, the independent scattering theory breakdowns due to the failure of the far-field approximation [25] and the non-negligible effects of shadowing and interference of scattered waves. Exact solution can be retrieved using microphysical approaches based on electromagnetic wave treatment. More details on them were reported in textbook by Mishchenko et al. [26]. However, when the characteristic size of the system is much larger than the wavelength, the microphysical approaches suffer computational limitations. As far as statistically isotropic and homogeneous dispersed media are concerned, the most common and experimentally verified approach has been to treat the dispersed medium as continuous and homogeneous, and to use the standard RTE with "effective radiative properties." This approach was proposed initially in the early 1980s and more particularly by Tien and co-workers, e.g. in [27–32], and few years later by Kamiuto [33], and Kaviany and Singh [34,35]. A more complete list of early investigations on effective radiative properties of densely packed systems can be found in [4,36]. Note that the terms "radiative properties with dependent scattering" has been often used (e.g. in [3,27–32]) to designate the effective radiative properties thus highlighting their difference with those predicted by the independent scattering theory. It is important to note that when the dispersed media are not statistically homogeneous and isotropic, the traditional RTE becomes questionable and the so-called Generalized radiative transfer equation is recommended [8–10,37].

Closely spaced particles do not scatter independently the radiation because: (i) the particles within an element volume obstruct each other creating thus shadowing effect; and (ii) the wave incident on each particle is affected by the waves scattered from close particles; as result, multiple scattering (in contrast to single scattering) takes place within the element volume. In the limit of large particles with respect to the wavelength, the shadowing and multiple scattering effects constitute the "dependent scattering effects" [33–35,38–40]. They need to be captured through the effective radiative properties enabling us to use the classical RTE. For packed beds consisting of statistically isotropic and homogeneous opaque spheres in a transparent gas, the radiative properties with dependent scattering effects were established by scaling the absorption and scattering coefficients predicted by the independent scattering theory by a depending scattering factor [35,38,40]. We will give more details about this factor later. The effective radiative properties of semi-transparent particle systems are more difficult to predict than those of opaque particle systems. A radiation bundle crossing a large semi-transparent particle travels a distance about the particle size. This distance across the particle has been referred