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Forced convection in a parallel-plate channel occupied by a nanofluid or a porous medium saturated by a nanofluid

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ABSTRACT

An analytical study is made of fully-developed laminar forced convection in a parallel-plate channel occupied by a nanofluid or by a porous medium saturated by a nanofluid, subject to uniform-flux boundary conditions. A model incorporating the effects of Brownian motion and thermophoresis is adopted. (Previous analytical studies using this model have been concerned with natural convection.) It is found that the combined effect of these two agencies is to reduce the Nusselt number.

1. Introduction

In recent years a large number of analytical studies of natural convection in nanofluids (suspensions of particles whose diameters are tens or hundreds nanometres) have been published, either in a fluid otherwise clear of solid material or in a porous medium, but the corresponding literature on forced convection is more sparse. In the case of a porous medium we are aware of just one published study, namely that by Maghrebi et al. [1], and that study was a numerical one. These authors investigated thermally developing forced convection in a parallel-plate channel.

For a fluid clear of solid material there have been several numerical as well as experimental investigations, and these have been reviewed by Dalkilic et al. [2]. A recent one is that by Rossi di Schio et al. [3]. However, as far as we know there has been just a single analytical published paper, namely that by Hung [4], who studied flow in microchannels. He considered just the variation of thermal conductivity, viscosity and heat capacity. On the other hand, Maghrebi et al. [1] employed the Buongiorno [5] model which incorporates the effects of Brownian motion and thermophoresis.

In the present paper we present an analytical study of fully-developed forced convection, using the Buongiorno model, in a parallel-plane channel with uniform heat flux on the boundaries. In the numerical work of Rossi di Schio et al. [3] the Buongiorno model was used, but these authors assumed streamwise variable temperature boundary conditions rather than constant flux ones.

2. Analysis

In the analysis that follows we use asterisks to denote variables that have dimensions. We chose Cartesian coordinates such that the boundary plates are at $z^* = \pm H$, so that H is the channel half-width. Then for the case of a nanofluid otherwise clear of solid material (for brevity we will use the phrase "clear fluid"), the velocity $\mathbf{v}^*$ has components $(U, 0, 0)$, where U is given by

$$U = \frac{3}{2} U_m (1 - z^2), \tag{1}$$

where U_m is the mean velocity. This is the well-known expression for plane-Poiseuille flow. For the case of a Darcy porous medium, we have slug flow, and

$$U = U_m. \tag{2}$$

We denote the temperature by T^* and the volumetric nanoparticle fraction by ϕ^*. At the boundaries we suppose that the heat flux into the porous medium or clear nanofluid is q'', a constant quantity. Since the flow is thermally developed, the first law of thermodynamics requires that

$$\frac{\partial T^*}{\partial x^*} = \frac{q''}{(\rho c)_f H U_m}. \tag{3}$$

Here $(\rho c)_f$ is the heat capacity of the nanofluid. The thermal energy and nanoparticle conservation equations are taken to be

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Nomenclature

c	nanofluid specific heat at constant pressure
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
H	half channel width
k_m	thermal conductivity of the porous medium
k_N	effective thermal conductivity of the nanofluid or porous medium
Le	Lewis number, defined in Eq. (8)
N_A	modified diffusivity ratio, defined in Eq. (8)
N_B	modified heat capacity ratio, defined in Eq. (8)
Pe	Péclet number, defined in Eq. (12)
q''	wall heat flux
t^*	time
t	dimensionless time, $t^*\alpha_m/\sigma H^2$
T^*	nanofluid temperature
T	dimensionless temperature
T_0	representative temperature, chosen to be the wall temperature
(u, v, w)	dimensionless Darcy velocity components, $(u^*, v^*, w^*)H/\alpha_m$
U_m	mean flow velocity
$\mathbf{v}$	fluid velocity
$\mathbf{v}^*$	dimensional fluid velocity, (u^*, v^*, w^*)
(x, y, z)	dimensionless Cartesian coordinates, defined in Eq. (6)
(x^*, y^*, z^*)	Cartesian coordinates

Greek symbols

α_N	thermal diffusivity of the nanofluid, $\frac{k_N}{(\rho c_P)_f}$
ε	porosity of the porous medium (set as unity for a clear fluid)
μ	viscosity of the fluid
ρ	fluid density
$(\rho c)_f$	heat capacity of the nanofluid
$(\rho c)_N$	effective heat capacity of the nanofluid or porous medium
$(\rho c)_p$	heat capacity of the nanoparticles
ρ_p	nanoparticle mass density
σ	heat capacity ratio, defined in Eq. (7)
ϕ^*	nanoparticle volume fraction
ϕ_0^*	reference value for the nanoparticle volume fraction
ϕ	relative nanoparticle volume fraction, $\frac{\phi^*-\phi_0^*}{\phi_0^*}$
Φ	function giving z-dependence of ϕ
θ	dimensionless temperature
Θ	function giving z-dependence of θ

Superscript

$*$	dimensional variable

$$(\rho c)_N \frac{\partial T^*}{\partial t^*} + (\rho c)_f \mathbf{v}^* \cdot \nabla^* T^* = k_m \nabla^{*2} T^* + \varepsilon(\rho c)_p \left[D_B \nabla^* \phi^* \cdot \nabla^* T^* + \left(\frac{D_T}{T_0}\right) \nabla^* T^* \cdot \nabla^* T^* \right], \quad (4)$$

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon} \mathbf{v}^* \cdot \nabla^* T^* = D_B \nabla^{*2} \phi^* + \left(\frac{D_T}{T_0}\right) \nabla^{*2} T^*. \quad (5)$$

Here t^* is the time, $(\rho c)_N$ is the heat capacity of the porous medium or clear nanofluid, k_m is the thermal conductivity of the porous medium, ε is the porosity of the porous medium (set as unity for a clear fluid), $(\rho c)_p$ is the heat capacity of the nanoparticles, D_B and D_T are the Brownian motion and thermophoresis coefficients, respectively, and T_0 is a representative temperature which we can choose, in the absence of any better alternative, to be the wall temperature, even though in the case of uniform flux boundary conditions this varies with the axial coordinate. We are following the classical treatment of forced convection given in textbooks such as Bejan [6].

We introduce dimensionless variables as follows. We define

$$(x, y, z) = (x^*, y^*, z^*)/H,$$
$$t = t^*\alpha_m/\sigma H^2, \quad (u, v, w) = (u^*, v^*, w^*)H/\alpha_m,$$
$$\phi = (\phi^* - \phi_0^*)/\phi_0^*, \quad T = (T^* - T_0)k_m/Hq'', \quad (6)$$

where ϕ_0^* is a reference scale for the nanopartical fraction and

$$\alpha_N = \frac{k_N}{(\rho c_P)_f}, \quad \sigma = \frac{(\rho c_P)_N}{(\rho c_P)_f}. \quad (7)$$

We also introduce the dimensionless parameters Le, N_A and N_B defined by

$$\mathrm{Le} = \frac{\alpha_N}{D_B}, \quad N_A = \frac{D_T}{D_B T_0}, \quad N_B = \frac{\varepsilon(\rho c)_p \phi_0^*}{(\rho c)_f}. \quad (8)$$

Then Eqs. (4) and (5) become

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + \frac{N_B}{\mathrm{Le}} \nabla \phi \cdot \nabla T + \frac{N_A N_B}{\mathrm{Le}} \nabla T \cdot \nabla T, \quad (9)$$

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} \mathbf{v} \cdot \nabla \phi = \frac{1}{\mathrm{Le}} \nabla^2 \phi + \frac{N_A}{\mathrm{Le}} \nabla^2 T. \quad (10)$$

The parameter Le is a Lewis number, N_A is a modified nanofluid diffusivity ratio and N_B is a modified nanofluid heat capacity ratio.

Eq. (3) becomes

$$\frac{\partial T}{\partial x} = \frac{1}{\mathrm{Pe}}, \quad (11)$$

where Pe is a Péclet number defined by

$$\mathrm{Pe} = \frac{HU_m}{\alpha_N}. \quad (12)$$

In this notation,

$$u = \mathrm{Pe} f(z), \quad (13)$$

where

$$f(z) = \begin{cases} 1 & \text{for a porous medium,} \\ \frac{3}{2}(1 - z^2) & \text{for a clear fluid.} \end{cases} \quad (14)$$

Now Eqs. (9) and (10) become, for a steady-state situation,

$$\nabla^2 T + \frac{N_B}{\mathrm{Le}} \nabla \phi \cdot \nabla T + \frac{N_A N_B}{\mathrm{Le}} \nabla T \cdot \nabla T = f(z), \quad (15)$$

$$\nabla^2 \phi + \frac{N_A}{\mathrm{Le}} \nabla^2 T - \mathrm{Le}\mathrm{Pe} f(z) \frac{\partial \phi}{\partial x} = 0. \quad (16)$$

We need to solve Eqs. (15) and (16) subject to the symmetry conditions

$$\frac{\partial T}{\partial z} = 0, \quad \frac{\partial \phi}{\partial z} = 0 \text{ at } z = 0, \quad (17a, b)$$

and the boundary conditions

$$T = \frac{x}{\mathrm{Pe}}, \quad \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \text{ at } z = 1. \quad (18a, b)$$