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Transition to instability of liquid–vapour front in a porous medium cooled from above

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ABSTRACT

The instability of the liquid–vapour front in a geothermal system with a cooling flux at the liquid boundary is investigated by introducing a small perturbation at the front. The mechanisms contributing to the stability and instability of such systems are analysed using a separate-phase model with a sharp interface between liquid and vapour. The governing equations representing incompressibility, Darcy's law and energy conservation for each phase are linearised about suitable base state and the stability of this state is investigated. A conditional expression for the critical modified Rayleigh number depending on the different physical parameters has been found. It has been shown that advection is not essential for instability, but it encourages the unstable behaviour.

1. Introduction

The interest in the investigation of liquid–vapour phase change problems arises from their wide range of applications, such as drying processes [1,2, pp. 397–409], geothermal systems [3–8], heat pipes [9], film boiling [10–12] and nuclear safety analysis [13]. The additional heat which is absorbed or released in these processes during the transformation of one phase to another phase is known as the latent heat of evaporation or condensation.

The transitions to instability at fluid–fluid interfaces are of great interest on account of their above mentioned applications. These instabilities can often occur at a phase change interface between liquid and vapour. In particular, we are interested in instabilities related to the Rayleigh–Taylor instability when they occur in porous media. The basic state in such configurations becomes unstable when the buoyancy forces overcome the stabilising effect of viscosity and diffusion. It happens for a critical value of the Rayleigh number, which is considered to be the key controlling parameter.

In the present study the instability of the liquid–vapour front in a porous medium with a cooling flux at the liquid boundary, is considered. The controlling dimensionless parameters of the flow and the heat transport are the modified Rayleigh number (R_3) and the cooling flux at the liquid boundary (Q_{liq}), respectively. The base temperature and velocity profiles with $|R_3| = 5$, the density ratio $R_1 = 0.0006$, the conductivity ratio $\kappa = 4$, $Q_{liq} = 1.2, 2, 4$, and the scaled length of the porous layer $x^* = 1$, are presented in Fig. 1. The stability of this base state is in question. It is important to note that Figure 1(b) does not shows the absolute pressure, but simply the difference between the absolute pressure at a given point and the pressure on the (upper) boundary.

In a geothermal context, despite of the strong buoyancy contrast between the liquid and vapour phases, the natural (basic) state is often found to be stable [14,15]. Straus & Schubert [16] carried out a linear stability analysis of the phase change front. In the basic (unperturbed) state, the fluid phases were considered to be static with the assumption that there is no net mass flux across the phase change interface; they showed that the phase change interface is gravitationally unstable for medium wavelength. Furthermore, if net mass flux is allowed through the interface then the stability of the system is permeability dependent. Later, Straus & Schubert [17] illustrated that a vapour dominated system can develop, if the phase change interface exists in a low permeability layer. Eastwood & Spanos [18] showed that if the phase change interface is sharp and there is a zero net mass flux across the interface then the system is unstable for long wavelength. In the case, when phase transition (net mass flux is allowed) is permitted then neutral stability can be predicted for a critical wave number.

Tsypkin & Il'ichev [19,20] categorized three different cases of transition to instability of the stationary vertical phase change flow under the condition that conduction dominates over advection. It was shown that if the interface is equidistant from the liquid and vapour boundaries then there is a spontaneous transition to instability (all wave numbers become unstable at the same value of the controlling parameter). The remaining two cases were: that the transition to instability occurs first at zero wave number if the

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Nomenclature

Latin

$\dot{m}$ mass flux
c_p specific heat
g acceleration due to gravity
H reciprocal of Stefan number
K permeability
k thermal conductivity
L thickness of the low permeable layer
l wave number
P pressure
q heat flux per unit area
S location parameter of the interface
T temperature
t time
x vertical coordinate
y horizontal coordinate

Greek symbols

ϵ perturbation parameter
κ thermal conductivities ratio
λ latent heat
μ dynamic viscosity
ν kinematic viscosity
ρ density
σ spectral parameter
σ^* asymptotic spectral parameter
Θ dimensionless temperature
υ fluid flow velocity
φ porosity

Dimensionless quantities

C specific heat ratio
E heat capacity ratio
R kinematic viscosity ratio
R_1 density ratio
R_2 dynamic viscosity ratio
R_3 modified Rayleigh number

Subscripts

L liquid boundary
liq liquid phase
m porous medium
ref reference quantity
S at the phase transition front
s porous skeleton
V vapour boundary
vap vapour phase
0 base state
1 perturbed state

Superscripts

min minimum
$*$ dimensionless quantity
0 base state
1 perturbed state

interface is near to the vapour boundary, whereas instability occurs first at infinite wave numbers if the interface is near to the liquid boundary. Il'ichev & Tsypkin [7] concluded that the most unstable mode of transition happens for zero wave number, when a water phase overlies an air-vapour mixture phase. Later on, Il'ichev & Tsypkin [21] studied the stability of water over steam with an advective-conductive basic state and showed that for an arbitrary value of the permeability a vapour dominated phase may exist and be stable. Most recently, Khan & Pritchard [14] carried out linear stability analysis of liquid–vapour fronts in porous media with various combinations of thermal boundary conditions. They showed that the liquid–vapour interface has multiple positions and reaffirmed that despite of heavier fluid above lighter fluid configuration, the basic state could be still stable. The main difference with the current study is that now we consider the simplest possible base state (no through flow) and explore the possibility for the better understanding of the medium-wave instabilities while focusing on advection with in the perturbed state. Despite the fact that the base state has no through flow but still the present model inherit all the physical parameters from the previous study.

In Section 2 we will present the mathematical model we employ. We will then (Section 3) consider a linear stability analysis of the steady state. The possible types of transition to instability, namely, transitions due to long- and short-wave perturbations are discussed (Section 4). In Section 5 we will present the comparison of the different modes of heat transport in the perturbed state. Finally, in Section 6 we will consider the physical interpretation and significance of our results.

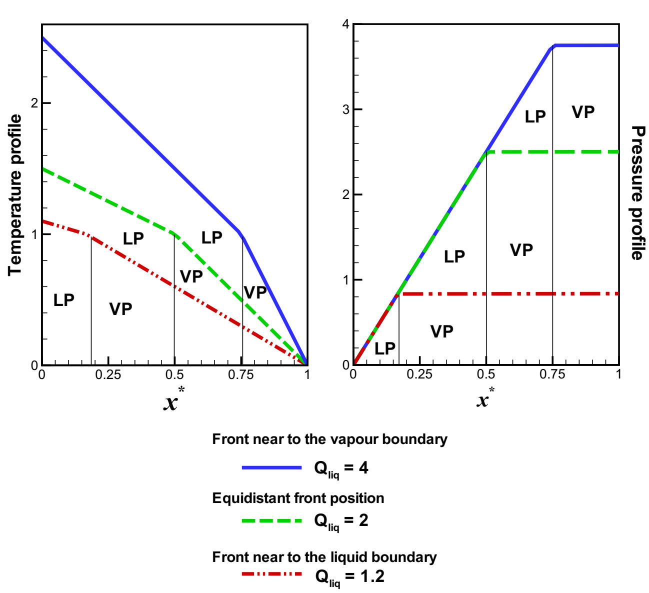

Fig. 1. Base temperature and velocity profiles, where LP and VP stands for the liquid and the vapour phase, respectively.

2. Mathematical model

We consider a porous layer of infinite extension bounded by two horizontal, much more permeable layers. The porous medium is considered to be uniform, isotropic and fully saturated with fluid. The upper and lower highly permeable layers are filled either with vapour and liquid, respectively or liquid and vapour, respectively (see Fig. 2). In the low-permeability layer there exists a phase change front which separates the liquid phase from the vapour phase. The liquid side is kept cool by imposing a cooling flux, whereas the vapour side is hot. The highly permeable layers will allow us to impose constant pressures at both sides of the low-permeability layer.