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Scaling properties of the equation for passive scalar transport in wall-bounded turbulent flows

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ABSTRACT

Data from direct numerical simulations (DNS) of fully-developed turbulent channel flows subjected to a constant surface heat-flux are used to explore the scaling behaviours admitted by the mean thermal energy equation. Following the framework of Wei et al. (2005) [1,2], the analysis employs a theory based on the magnitude ordering of terms in the mean thermal energy equation of wall-bounded turbulent heat transfer. A four layer thermal structure has been identified from the leading order terms in the mean energy equation. A review of the limitations of traditional and existing scaling of mean temperature and turbulent heat flux is conducted. The possibilities of a new scaling approach with the introduction of generalized thermal length scale are discussed within the context of the four-layer framework. This methodology generally seeks to determine the invariant form(s) admitted by the relevant equation. Investigation of normalized statistical quantities applicable to inner, outer and intermediate regions of the flow, whose properties are dependent on a small parameter that is a function of either Reynolds number or both Reynolds and Prandtl numbers, shows inconsistencies between the normalizations on the different subdomains. Although the present scaling approach successfully explores the generalized properties of intermediate layer, issues pertaining to simultaneously and self-consistently reconciling the inner and intermediate normalizations remain unresolved.

1. Introduction

Wall-bounded turbulent flows are present in numerous industrial, technological, aerospace and naval applications that involve heat and mass transport. The knowledge of the mean temperature profile is generally essential, and a number of approaches have been attempted to predict the variation of this scalar field over the flow domain. Based on the Reynolds analogy between momentum and scalar transport, many researchers have employed approaches that effectively assume 'the law of wall' [3–5]. This approach supposes that the mean temperature and turbulent heat flux profiles become invariant when the viscous or inner scaled distance from the wall is employed. Conveniently, one may then apply this form of the 'Reynolds analogy' to relate the eddy viscosity to the eddy thermal diffusivity. A brief review of the many variations of this approach are listed by Dhotre and Joshi [6]. Such approaches also naturally embrace the use of higher order closures for the Reynolds averaged momentum and heat balance equations. Although these kinds of models are fast and amenable to use at very high Reynolds and Prandtl numbers, the correct estimation of mean quantities critically depends on the accurate determination of the appropriate normalizations and the length, velocity and temperature scales they employ. Earlier investigations showed that the normalized mean temperature only exhibits slight variations due to the Reynolds number [7,8]. Temperature profiles, however, are seen to change much more rapidly with varying Prandtl number, both in turbulent channel [7] and pipe [9] flow. To date, the combined effects of Reynolds and Prandtl numbers have not been systematically investigated in the context of the underlying transport equation. It is important, however, to understand how the thermal transport equation can be cast into invariant forms that properly reflect the dominant physical mechanism, as this reveals the effects of the governing parameters on the thermal field statistics.

Traditional representations of temperature and turbulent heat flux profiles generally employ either inner or outer normalizations. These normalizations, however, fail to provide invariant profiles as the relevant non-dimensional parameters are varied [7–25]. Moreover, neither of these normalization are successful in

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the vicinity of the peak turbulent heat flux profiles. Inner normalization of the mean temperature uses the so-called friction temperature, and the wall distance is normalized by the friction velocity and the kinematic viscosity. This normalization, however, is traditionally relevant over a small region adjacent to the wall, the conductive sublayer [4], whose width varies as a function of Prandtl number. Furthermore, the data from the logarithmic layer for temperature exhibit different mean temperature profiles as a function of both Reynolds and Prandtl numbers. This range of phenomena is richer than exhibited by the momentum field. It arises from the additional parameter, Prandtl number.

In order to understand the underlying physics of heat transfer in turbulent flows for moderate to high Reynolds and Prandtl numbers, dimensional and similarity analysis play central roles. In this regard, the literature is extensive, and thus here we only discuss a subset of recent findings. Wang et al. [5] introduced the temperature scaling for forced convection turbulent boundary layers using a variant of the similarity theory by George and Castillo [26]. A power law was found for the temperature profile in an intermediate region, and this melds into a composite profile in the wake and near-wall regions. Apart from dimensional analysis or similarity analysis, Churchill and Chan [27] and Churchill et al. [28] introduced a new approach by proposing an algebraic model to predict the mean temperature profile from a knowledge of the velocity profile and the turbulent Prandtl number. Using the model of Churchill and Chan [27] and Churchill et al. [28], Le and Papavassiliou [29] developed a temperature profile for low Reynolds number turbulent flow. But they pointed out the limitation of the theoretical predictions by Churchill and co-workers at very high Prandtl numbers. Marati et al. [30] derived the symmetry invariant mean profiles for a passive scalar in wall-bounded turbulent flow based on the symmetry properties of the Navier–Stokes equation and the energy equation. Their results showed the validation of the well-known logarithmic laws as well as interpreted linear, algebraic and exponential profiles in different physical regimes.

Building upon his initial studies indicating the existence of an intermediate layer (mesolayer), Afzal [31] employed a different approach to investigate the properties of the mean momentum and thermal balance in fully developed turbulent channel flow having both smooth and transitionally rough surfaces. Seena and Afzal [32] proposed a power law temperature distribution for a fully developed turbulent channel flow for large Peclet numbers (product of Reynolds and Prandtl numbers). They supposed that both the mean turbulent flow and thermal fields were divided into inner and outer layers. The matching of the velocity profile by the Isakson–Millikan–Kolmogorov hypothesis [33–35] led to a power law velocity profile [36,37], in addition to the traditional log laws. Similar analyses were used to deduce a power law temperature profile [32], which was proposed to be equivalent to the log-law temperature profile for large Peclet numbers. Seena and Afzal [38] also studied the scaling properties of the intermediate layer in a fully developed turbulent channel flow by employing the method of matched asymptotic expansions. They proposed a half-defect velocity law and a half temperature defect law in association with the intermediate layer. Their prediction of Reynolds shear stress and Reynolds heat flux profiles in the intermediate layer show good agreement with available experimental and DNS data. Moreover, by assuming the existence of overlap layers Seena et al. [39] constructed a closure model that leads to a series of logarithmic functions of the mesolayer variable for Reynolds shear stress and Reynolds heat flux profiles.

Herein we take a different approach to study the scaling properties admitted by the mean thermal energy equation. This framework only relies on the magnitude ordering of the terms in the mean energy equation, and thus does not invoke additional assumptions or resort to the use of a closure model. Recent analyses of turbulent wall bounded flow for both pipe and channel [1,40–43] indicate that many of the statistical properties of these flows are similar, even though they possess different geometric configurations. Notably, analyses of the mean momentum equation can be directly employed to explore the underlying physics and scaling of the dependent variables in that equation. Wei et al. [1] introduced a generic first-principles framework to characterize the four layer regime in wall bounded flows, an extension of which leads to a mesoscaling of Reynolds shear stress [43] and mean velocity field [44] in turbulent channel flows. The limiting value of Reynolds number at which the four layer magnitude orderings are first established has been investigated for channel flows by Elsnab et al. [45]. However, the onset of four-layer regime for thermal field is not yet well characterized, as it is a function of both Reynolds and Prandtl numbers. In fact, as shown herein, a number of conditions depending on the magnitude of Reynolds and Prandtl numbers factor into determining the onset of the four-layer thermal structure.

An important observation obtained from the mean momentum balance theory [46,47] is the existence of a hierarchy of scaling layers with each having an analytically well-defined characteristic length. The conditions for logarithmic dependence of the mean velocity profile were explored by using this approach [48,49]. The analogous method was subsequently applied to channel flow heat transfer by Wei et al. [2]. This effort revealed a qualitative characterization of the four layer regions, Peclet number dependence of the scaling of temperature, and the conditions associated with the existence of the logarithmic mean temperature profile. However, a more comprehensive elucidation of the scaling behaviours of the mean energy equation is still lacking, and this motivates the present effort.

Multiscale analyses are used herein to clarify the scaling properties admitted by the mean energy equation. In order to describe mean flow structure properly, a length scale intermediate to the traditional inner and outer scales is necessary. According to the present theory, the transition from inner to outer scaling physically takes place owing to a balance breaking and exchange of the leading order heat transport mechanisms as a function of scale. This underlies the existence of an intermediate region between inner and outer layers (thermal mesolayer) where, in the mean, all terms in the energy equation are of equal order [2]. In order to gain a better understanding of the possibilities for generating invariant profiles of the mean temperature and turbulent heat flux, the current investigation exploits the properties of four distinct balance layers in a magnitude ordering and scaling analysis of the mean energy equation. The analyses primarily employ existing DNS data sets of Kawamura and co-workers [17,20,50].

2. Mean momentum layer structure

To provide a context for the heat transfer problem, it is useful to briefly review the four layer structure associated with the mean momentum balance. The relative magnitude of the terms in the Reynolds-averaged Navier–Stokes equations are used to define the layer properties. This fundamentally differs from the traditional four layer structure for turbulent channel flows [51–53]; namely the viscous sublayer ($y^+ = yu_\tau/\nu < 5$, where y is the wall-normal distance, ν is the kinematic viscosity and u_τ is the friction velocity), the buffer layer ($5 \leqslant y^+ \leqslant 30$), the inertial (or classical logarithmic layer, $30 \leqslant y^+ \lesssim 0.15Re_\tau$, where Re_τ is the Kármán number, $Re_\tau = u_\tau\delta/\nu$) and the outer boundary layer or wake layer ($0.15 \leqslant y/\delta \lesssim 1.0$). It also fundamentally differs from the structure proposed by Wosnik et al. [54]. They divided the flow into the main 'viscous sublayer', 'overlap' and 'outer' regions. The near-wall region, where $0 < y^+ < 30$, was composed of the linear viscous