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Analysis of heat transfer and stability of magnetohydrodynamic natural convection above a horizontal plate with heat flux boundary condition

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ABSTRACT

A boundary layer analysis is performed for the steady laminar natural convection of an electrically conducting viscous incompressible fluid above a horizontal plate in the presence of a transverse magnetic field. The governing boundary layer equations are transformed to a set of non-linear ordinary differential equations which are then solved numerically using the shooting method. The effects of Prandtl number Pr and magnetic field parameter ζ on the velocity profiles, temperature profiles, skin friction coefficients and heat transfer coefficients are presented and discussed. A linear stability analysis based on the parallel flow model is also carried out to study the vortex instability of such flow. The present results indicate that the skin friction coefficient c_{fx} decreases whereas the local Nusselt number Nu_x increases with increase in Pr if ζ is held constant. On the other hand, both the skin friction coefficient and Nusselt number decrease with increase in ζ for a fluid with constant Pr. Generic correlations for Nu_x and c_{fx} have been developed in terms of Pr and ζ. A generic relation for magnetohydrodynamic free convection flow, equivalent to the Reynolds analogy for forced convection, has also been formulated.

1. Introduction

The study of magnetohydrodynamic free convection fluid flow and heat transfer of an electrically conducting, viscous fluid past a heated semi-infinite plate has attracted considerable research interest due to its relevance in several scientific fields, for instance in cosmic fluid dynamics, geophysical fluid dynamics, metallurgical fluid dynamics, re-entry aerothermodynamics, nuclear engineering, chemical vapor deposition reactors and MHD power generation [1–4]. In light of these applications, the natural convection boundary layer flow of an electrically conducting fluid over a semi-infinite heated vertical plate has been studied extensively by many researchers.

Gupta [1] developed similarity solutions for MHD free convection flow over an isothermal vertical surface when the imposed magnetic field (acting perpendicular to the plate) varies as $x^{-1/4}$, where x is the coordinate measured along the length of the plate. The similarity equations were solved using the Karman–Pohlhausen integral method. Sparrow and Cess [2] considered the effect of MHD free convection heat transfer over an isothermal vertical plate under the action of a uniform imposed magnetic field applied normal to the plate surface. Cramer [3] examined the influence of transverse magnetic field on laminar natural convection flow of low Prandtl number fluids over a vertical plate for varying plate temperature. Lykoudis [4] investigated natural convection of an electrically conducting fluid in the presence of a magnetic field acting normal to the vertical plate and formulated asymptotic solutions for the following three cases: (a) high Prandtl number and small magnetic field parameter, (b) high magnetic field parameter regardless of Prandtl number and (c) low Prandtl number. Singh and Cowling [5] have studied the natural convection boundary layer flow of an electrically conducting fluid over a heated vertical plate in the presence of a strong cross magnetic field. Their analysis reveals that regardless of the strength of the magnetic field, a region will always exist in the locality of the leading edge of the plate where the effect of electromagnetic forces can be neglected.

Riley [6] developed a solution for MHD natural convection boundary layer flow over a heated vertical plate in the presence of a strong cross magnetic field applying the matched asymptotic expansion method. Later, Kuiken [7] re-examined the same problem posed by Riley [6] and obtained solution of the boundary layer equations using singular perturbation technique. The analysis is carried out for fluids with Prandtl number ranging from 0 to 1. Wilks [8] analyzed free convection flow of an electrically conducting, non-magnetic fluid over a heated vertical plate in the presence of a strong magnetic field applied normal to the plate and obtained numerical solutions for various values of the parameter ξ which represents the relative magnitude of magnetic and buoyancy forces at varying locations along the plate. Wilks and Hunt [9] reconsidered the same problem posed above by taking into account

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Nomenclature

B	magnetic field strength
D^n	differential operator ($=d^n/d\eta^n$)
f	reduced stream function defined by Eq. (10)
g	gravitational acceleration
Gr_x^*, Gr_L^*	Grashof numbers defined, respectively, as $\frac{g\beta q_w x^4}{k\nu^2}$ and $\frac{g\beta q_w L^4}{k\nu^2}$
h	heat transfer coefficient defined as $q_w/(T_w - T_\infty)$
k	thermal conductivity of the fluid
L	reference length of the plate in x direction
Nu_x	local Nusselt number, hx/k
p	static pressure
p_∞	static pressure in the undisturbed fluid
Pr	Prandtl number, ν/α
q	prescribed heat flux
T	static temperature
T_∞	static temperature in the undisturbed fluid
u	velocity component in the x direction
$\hat{u}$	dimensionless amplitude function of velocity disturbance in x direction
u_0	reference velocity in the x direction
v	velocity component in the y direction
$\hat{v}$	dimensionless amplitude function of velocity disturbance in y direction
$\vec{v}$	velocity vector
x	horizontal coordinate
y	vertical coordinate

Greek symbols

α	thermal diffusivity
$\hat{\alpha}$	dimensionless wave number of disturbances
β	coefficient of thermal expansion at the reference temperature
ζ	magnetic field parameter defined as $\sigma \frac{B^2 x^2}{\mu} \frac{1}{(Gr_x^*)^{1/3}}$
η	similarity variable
θ	non-dimensional temperature defined by Eq. (10)
$\hat{\theta}$	dimensionless amplitude function of temperature disturbance
μ	dynamic viscosity
μ_e	magnetic permeability
ν	kinematic viscosity
ρ	density of fluid
σ	electrical conductivity of the fluid
ϕ	reduced pressure difference defined by Eq. (10)
ψ	stream function defined by Eq. (10)

Subscripts

c	critical value
w	condition at the wall
∞	condition in undisturbed fluid

Superscript

$'$	differentiation with respect to η

uniform heat flux at the plate and obtained a full numerical solution. Gupta [10] developed integral solutions of laminar free convection flow of an electrically conducting fluid past a vertical plate when the plate is either subjected to a uniform heat flux or variable wall temperature.

Combined effects of heat and mass transfer on steady, laminar free convection flow of an electrically conducting fluid over a semi-infinite vertical plate in the presence of a transverse magnetic field can be found in the works of Gupta [11] and Soundalgekar [12]. Takhar and Soundalgekar [13] have studied the effects of viscous and Joule heating on the MHD free convection flow past an isothermal vertical plate using the series expansion method. Hossain [14] investigated the combined effects of viscous dissipation and Joule heating on the MHD natural convection flow of an electrically conducting fluid over a semi-infinite vertical surface when the surface temperature varies linearly with the distance from the leading edge. Chen [15] analyzed magnetohydrodynamic natural convection flow over a permeable inclined surface with variable wall temperature and concentration.

The above discussion shows that the MHD free convection flow past a semi-infinite vertical plate has been thoroughly investigated. However, the equally interesting problem of MHD natural convection flow over a horizontal plate has not received much attention. The present authors could locate only four papers – Gupta [16], Singh and Cremers [17], Singh [18], and Singh [19] – in the published literature that considered the combined effects of buoyancy and magnetic field on free convection heat transfer over a semi-infinite horizontal plate. Interestingly, the number of studies of laminar free convection heat transfer over semi-infinite horizontal plate (i.e. without MHD) is also limited, a few examples being the works of Stewartson [20], Chen et al. [21], Lin et al. [22] and Samanta and Guha [23].

Gupta [16], Singh and Cremers [17] and Singh [18] considered the effects of steady laminar MHD free convection from an isothermal horizontal plate using an integral analysis. Singh [19] attempted to develop a similarity solution for steady laminar MHD natural convection boundary layer flow along a semi-infinite horizontal flat plate maintained at uniform surface temperature but his analysis contains several inaccuracies. For example, in the analysis of Singh [19], the Grashof number for laminar natural convection is assumed to be 1. The magnetic field parameter ζ obtained after non-dimensionalization of the boundary layer equations is also inaccurate. Samanta and Guha [23] derived the boundary layer equations from first principles and showed that the equations are valid in the limit of large Grashof number when the buoyancy force dominates over viscous forces. Cheng and Kim [24] experimentally studied vortex instability of natural convection above an isothermal horizontal plate and concluded that transition from laminar to turbulent flow occurs at a critical Grashof number $\sim 10^7$. In a recent study, Guha and Samanta [25] developed closed-form analytical solutions for laminar natural convection above horizontal plates. CFD simulations for magnetohydrodynamic natural convection flow have been discussed in [26–28].

The purpose of the present investigation is to develop a similarity solution and to study the effects of a transversely applied magnetic field on laminar free convection of an electrically-conducting fluid above a semi-infinite horizontal plate subjected to constant surface heat flux. The stability of such flow has also been investigated.

2. Mathematical formulation for the base flow

Consider the steady laminar two-dimensional MHD natural convection boundary layer flow of an electrically conducting fluid over a semi-infinite horizontal plate in presence of a transverse magnetic field. The plate is electrically non-conducting and subjected to a constant heat flux q_w. The quiescent ambient fluid is maintained at a uniform temperature T_∞ and pressure p_∞. The physical model and coordinate system is presented in Fig. 1.

The natural convection boundary layer above a heated horizontal plate is formed indirectly because of an induced pressure