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Thermocapillary flow of thin liquid film over a porous stretching sheet in presence of suction/injection

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ABSTRACT

The thin film flow over a porous stretching sheet has been studied in presence of suction/injection under the assumption of uniform initial film thickness. It is also considered that the sheet is either heating or cooling along the direction of stretching. The rate of film thinning decreases with increasing the thermocapillary parameter when the sheet is heated and the opposite phenomena is observed for the sheet is cooled. The film thickness decreases with the increasing values of porosity parameter and suction velocity whereas it decreases with the increasing injection velocity. The temperature at a fixed height of the film decreases with the increasing values of porosity parameter and injection velocity, but it increases with increasing suction velocity when the sheet is heating. The opposite behaviour is observed when the sheet is cooling.

1. Introduction

The flow and heat transfer of a thin viscous liquid film over a stretching sheet is important for understanding and designing of various heat exchangers and chemical processing equipment, reactor fluidization etc. In a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a filament or sheet, which is then solidified through rapid quenching or gradual cooling by direct contact with water or chilled metal rolls. In fact, stretching imparts a unidirectional orientation to the extrudate, thereby improving its mechanical properties as the quality of the final product greatly depends on the rate of cooling. Crane [1] first studied a steady two-dimensional boundary layer flow caused by the stretching of a sheet that moves in its own plane with a velocity which varies linearly with the distance from a fixed point on the sheet and gave an exact similarity solution. Due to the practical applications of the stretching sheet flow problem, the work of Crane [1] was subsequently extended by many researchers either by considering the effects of rotation, heat and mass transfer, chemical reaction, MHD, non-Newtonian fluid or different possible combinations of these above effects (see [2–8]).

The study of hydrodynamic flow and heat transfer over a porous stretching sheet has received adequate attention due to its vast applications such as, geothermal extraction, storage of radioactive nuclear waste materials, cooling of electronic components, food processing, casting and welding of manufacturing processes etc. Cheng and Minkowycz [9] and Cheng [10] studied the problem of free convection about a vertical impermeable flat plate in a saturated porous medium. The boundary layer flow of viscous liquid through the porous stretching sheet are investigated by Abel and Veena [11], Elbasbeshy and Bazid [12], Ali and Mehmood [13], and Abel et al. [14]. Where as, the effects of suction or injection on the boundary layer flows due to a stretching of the wall has been analyzed by Erickson et al. [15], Gupta and Gupta [16], Chen and Char [17], Elbasbeshy and Bazid [12].

Needless to say that in all these studies, boundary layer equation is considered and the boundary conditions are prescribed at the sheet and on the fluid outside the boundary layer at infinity. Imposition of similarity transformation reduces the system to a set of ODEs, which are then solved either analytically or numerically. Wang [18] first studied the flow of a thin liquid film over a unsteady stretching sheet. In this study, he used a special type of similarity transformation to reduce the boundary-layer equations into a set of nonlinear ODE and then solved numerically. Using this special type of similarity transformation, Andersson et al. [19,20] and Chen [23] extended the study of unsteady stretching sheet flow of a liquid film to the case of power law fluid, heat transfer. Liu and Andersson [21] extended the problem considered by Andersson et al. [20] in the case of more general form of the prescribed surface temperature variation. Dandapat et al. [22] studied the effects of thermocapillarity on the flow of thin liquid film over an unsteady stretching sheet. Wang [24] presented exact analytical solutions for the momentum and heat transfer within an unsteady thin liquid film over a stretching sheet. Nandeppanavar et al. [25] studied the combine effects of viscous dissipation, non-uniform heat source/sink, magnetic field and

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thermal radiation on the flow and heat transfer of a thin liquid film over an unsteady stretching surface.

It is to be mentioned here that the study of unsteady flow due to the stretching of a sheet has not yet received adequate attention when the liquid film either lies or coincide with the fluid boundary layer thickness. If the thickness of the liquid film either coincides or lies within the boundary layer thickness then one needs to consider the full set of Navier–Stokes equations to study such flow problem. Recently Dandapat et al. [26] and Dandapat and Maity [27] have studied the development of thin liquid film over an unsteady stretching sheet by considering the full set of Navier–Stokes equations with non-planer film surface and able to show that the final film thickness neither depends on the type of initial distribution of the liquid nor the initial amount of liquid deposited over the stretching sheet. Depending on the finding of Dandapat and Maity [27], Dandapat et al. [28] have studied the unsteady film development over a stretching sheet under the assumption of uniform initial film thickness.

To the best of our knowledge, the study of thin liquid film development over a porous stretching sheet by considering full Navier–Stokes equations has been not reported yet. In this article, we are interested to study the flow and heat transfer of a thin liquid film over an unsteady porous stretching sheet in presence of suction or injection by considering full set of momentum equations. We assumed that the initially deposited liquid film over the stretching sheet is planer and remain planer throughout entire process of stretching as well as film thinning. It is also assume that the sheet is either heating or cooling along the direction of stretching.

2. Mathematical formulation

Consider the unsteady flow of a thin liquid film of uniform thickness h_0 over a porous stretching sheet as shown in Fig. 1. The x-axis is chosen along the plane of the sheet and z-axis is taken normal to the plane. The surface $z = 0$ starts stretching impulsively from the rest with the velocity ax, a being constant with dimension of $[time]^{-1}$. The stretching sheet is either heating or cooling along the x-direction. The porous medium is assumed to be constant permeability $k'(>0)$ and the porosity $\phi(0 < \phi < 1)$, the effects of pores on the velocity field obey the Darcy's law $\nabla p = -\frac{\nu\phi}{k'}\mathbf{V}$ is given by Neild and Beijan [29]. Let $\mathbf{V} = (u, w)$ and T are the velocity vector and temperature of the liquid, respectively. The governing set of equations are

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\mathbf{V}_t + (\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p/\rho + \nu\nabla^2\mathbf{V} - \frac{\nu\phi}{k'}\mathbf{V}, \tag{2}$$

$$\rho C_p[T_t + (V \cdot \nabla)T] = k\nabla^2 T, \tag{3}$$

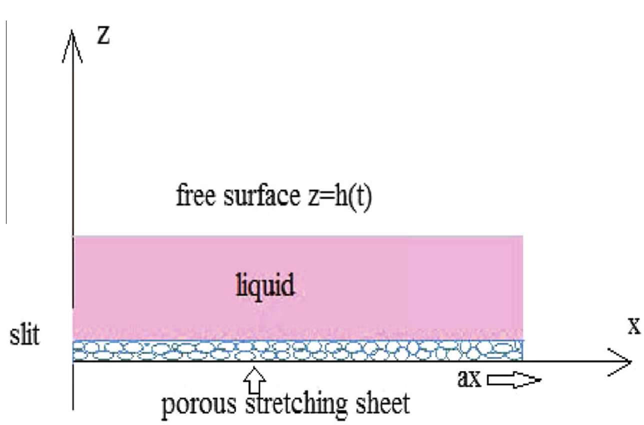

Fig. 1. Schematic diagram.

where ρ, ν, C_p and k are the density, kinematic viscosity, heat capacity at constant pressure and thermal conductivity of the liquid, respectively.

The corresponding boundary conditions are as follows.

On the plane of the sheet at $z = 0$,

$$u = ax, \quad w = -W_s, \quad T = T_0 - \lambda\frac{x^2}{2}T_1, \tag{4}$$

where T_0 and T_1 are positive constants and the velocity W_s is taken to be positive or negative for the suction or injection at the porous stretching wall. Here, the term heating or cooling is used to refer the situation of increasing or decreasing of the temperature along the direction of stretching respectively. The negative or positive values of λ represent the cases of heating or cooling along the stretching direction, respectively.

At the free surface $z = h(t)$,

$$p_a - p + 2\mu\frac{\partial w}{\partial z} = 0, \tag{5}$$

$$\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{\partial \sigma}{\partial T}\frac{\partial T}{\partial x}, \tag{6}$$

$$\frac{\partial T}{\partial z} = 0, \tag{7}$$

$$\frac{dh}{dt} = w, \tag{8}$$

where p_a and μ are the atmospheric pressure and dynamic viscosity of the liquid, respectively. Eqs. (5) and (6) denote respectively, the vanishing of the normal stress at the interface and shear stress is balanced by the thermal stress at the free surface. Eq. (7) represents that the heat flux at the free surface to be vanish, i.e., the free surface to be adiabatic. σ is the surface tension which varies linearly with the temperature as $\sigma = \sigma_0[1 - \gamma(T - T_0)]$ (see [22,30]). For the most of the liquids, surface tension decreases with temperature, i.e., γ is a positive constant. Eq. (8) represents the kinematic condition at the free surface.

The initial conditions at time $t = 0$ are

$$u = 0, \quad w = 0, \quad h(0) = h_0, \quad T = T_0. \tag{9}$$

Now we introduce the following similarity variables (see [30–32])

$$u(x, z, t) = xF(z, t), \quad w(x, z, t) = W(z, t), \tag{10}$$

$$p(x, z, t) = -\frac{x^2}{2}A(z, t) + B(z, t), \tag{11}$$

$$T(x, z, t) = T_0 - \lambda\frac{x^2}{2}M(z, t) - \lambda N(z, t), \tag{12}$$

where functions $M(z, t)$ and $N(z, t)$ appearing in 12 are clearly compatible with the temperature boundary condition given by (4). It is assumed that Eq. (12) holds for large but finite value of x so that T can never tend to ∞ or $-\infty$. Substituting (10)–(12) into the system of Eqs. (1)–(3) and equating the like order terms of x from both sides, we have

$$F + \frac{\partial W}{\partial z} = 0, \tag{13}$$

$$\frac{\partial F}{\partial t} + F^2 + W\frac{\partial F}{\partial z} = \frac{A}{\rho} + \nu\frac{\partial^2 F}{\partial z^2} - \frac{\nu\phi}{k'}F, \tag{14}$$

$$\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial z} = -\frac{1}{\rho}\frac{\partial B}{\partial z} + \nu\frac{\partial^2 W}{\partial z^2} - \frac{\nu\phi}{k'}W, \tag{15}$$

$$\frac{\partial A}{\partial z} = 0, \tag{16}$$

$$\rho C_p\left[\frac{\partial M}{\partial t} + 2FM + W\frac{\partial M}{\partial z}\right] = k\frac{\partial^2 M}{\partial z^2}, \tag{17}$$

$$\rho C_p\left[\frac{\partial N}{\partial t} + W\frac{\partial N}{\partial z}\right] = k\left[M + \frac{\partial^2 N}{\partial z^2}\right]. \tag{18}$$