International Journal of Heat and Mass Transfer 70 (2014) 864–874

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A coupled lattice Boltzmann and finite volume method for natural convection simulation

Zheng Li [a,b], Mo Yang [b], Yuwen Zhang [a,*]

[a] Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA

[b] College of Energy and Power Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China

ARTICLE INFO

Article history:
Received 21 September 2013
Received in revised form 26 November 2013
Accepted 26 November 2013
Available online 20 December 2013

Keywords:
Nonequilibrium extrapolation scheme
Lattice Boltzmann method
Finite volume method
Natural convection

ABSTRACT

A coupled lattice Boltzmann and finite volume method is proposed to solve natural convection in a differentially heated squared enclosure. The computational domain is divided into two subdomains and a message passing zone is between them. The velocity and temperature fields are respectively solved using D2Q9 and D2Q5 models in lattice Boltzmann method (LBM) while SIMPLE algorithm is applied to the finite volume method (FVM). The velocity and temperature information transfers are fulfilled by a non-equilibrium extrapolation scheme. Pure FVM, pure LBM, and the coupled method with two different geometric settings are applied to solve the natural convection with different Rayleigh numbers. The results obtained from the coupled method agreed with those from pure FVM and LBM very well.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

The fluid flow and heat transfer problems encountered in engineering applications span into different scales and there are different numerical methods for different scales. Molecular dynamics (MD) [1] can be applied to solve nano- and microscale problems while lattice Boltzmann method (LBM) is a typical mesoscopic scale method [2]. Finite volume method (FVM), on the other hand, is suitable for solving the macroscale problems [3]. The growing multiscale problems must be solved with multiscale method since there is no method that is suitable for all the scales. There exist research works about the multiscale methods in the literature, e.g., MD-LBM [4–6], MD-FVM [7–9] and LBM-FVM [10–12].

There are several models for the fluid flow and heat transfer problems in LBM. He et al. proposed the internal energy function by relating internal energy with the kinetic energy of particle for the incompressible fluid flow and heat transfer [13]. Then its simplified thermal LBM model was advanced by Peng et al. [14] while Guo et al. introduced a coupled lattice BGK model based on Boussinesq assumption [15]. The D2Q9 and D2Q5 models were respectively used to solve the velocity and temperature fields [16]. Meanwhile, Semi-implicit Method for Pressure Linked Equation (SIMPLE) [17] is one of the most frequently used algorithms in FVM. The D2Q9 and D2Q5 model were respectively used to solve the velocity and temperature fields [16]. Meanwhile, Semi-Implicitly Method for Pressure Linked Equation (SIMPLE) [17] is one of the most frequently used algorithms in FVM.

The objective of this paper is to combine the LBM and FVM as a coupled method to solve the fluid flow and heat transfer problem. The total computational domain is divided into FVM and LBM zones with a message passing zone between them. So there is an artificial boundary for each sub-domain. It is necessary to obtain the artificial information from the other sub-domain to fulfill the coupled method. For the fluid flow and heat transfer simulation, the problems under consideration are described using the macroscopic variables, such as velocity, temperature, pressure and density. And these variables in LBM are based on the density and energy distributions results while they can be solved directly in FVM. Meanwhile macroscopic variables can be transferred from the density and energy distributions results directly. So it is quite straightforward to obtain the FVM artificial boundary information from the inner nodes in the LBM zone. But macroscopic variables are not enough to obtain the corresponding density and energy distributions results.

Latt [18] and Latt et al. [19] coupled LBM and finite difference method (FDM) for the pure fluid flow with the first-order expansion of the lattice Boltzmann equation. However, the FDM itself has the limitation when solving problems with complex computational domain [3]. This shortfall restricts the development of LBM-FDM because the one of the most attractive advantages of LBM is its suitability to solve the problems in complex computational domain. Luan et al. [20] solved natural convection using the LBM-FVM with the general reconstruction operator [21]and obtained persuasive results. However general reconstruction

* Corresponding author. Tel.: +1 5738846936.
E-mail address: zhangyu@missouri.edu (Y. Zhang).

0017-9310/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.11.077

Nomenclature

c	lattice speed
c_s	speed of sound
$\boldsymbol{e}_i$	particle speed
F	body force
f_i	density distribution
$\boldsymbol{g}$	gravity acceleration (m/s^2)
$\boldsymbol{G}$	effective gravitational acceleration (m/s^2)
g_i	energy distribution
H	height of the cavity (m)
Ma	Mach number
Nu	Nusselt number
p	pressure (N/m^2)
P	nondimensional pressure
Pr	Prandtl number
Ra	Rayleigh number
t	time (s)
T	temperature (K)
u	horizontal velocity (m/s)
U	nondimensional horizontal velocity
v	vertical velocity (m/s)
V	nondimensional vertical velocity
$\boldsymbol{V}$	velocity

Greek Symbols

α	thermal diffusivity (m^2/s)
β	volume expansion coefficient of the fluid (K^{-1})
θ	nondimensional temperature
μ	viscosity (N s/m^2)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
τ	nondimensional time
τ_v	relaxation time for velocity
τ_T	relaxation time for energy
ω_i	value factor for velocity
ω_i^T	value factor for energy

operator is newly proposed to fulfill the combine method which means more validations are needed for the general reconstruction operator itself. The nonequilibrium scheme [15] is a valid LBM boundary condition that was reported to have the second order accuracy. This scheme is applied to obtain the density and energy distributions on the LBM zone artificial boundary using the results in FVM zone in this paper. The LBM is solving a compressible problem while FVM is based on incompressible assumption. Then a pressure based correction method is applied to transfer density from an incompressible domain to a compressible domain. Then natural convections in a squared enclosure with different Rayleigh numbers are solved using the coupled method and the results are compared with those obtained from pure LBM and pure FVM for validation of the coupled method.

2. Thermal LBM model

Two distribution functions are selected for the fluid flow and heat transfer in LBM. The density and energy distributions are represented by f_i and g_i, which are related by the buoyancy force. For the velocity field, D2Q9 model is preferred. There are nine local particle velocities on each computing node as shown in Fig. 1. These velocities are given by

$$\boldsymbol{e}_i = \begin{cases} (0,0) & i=1 \\ c\left(-\cos\frac{i\pi}{2}, -\sin\frac{i\pi}{2}\right) & i=2,3,4,5 \\ \sqrt{2}c\left(-\cos\frac{(2i+1)\pi}{4}, -\sin\frac{(2i+1)\pi}{4}\right) & i=6,7,8,9 \end{cases} \tag{1}$$

where c is the lattice speed.

The density and momentum can be obtained by

$$\rho = \sum_{i=1}^{9} f_i \tag{2}$$

$$\rho \boldsymbol{V} = \sum_{i=1}^{9} \boldsymbol{e}_i f_i \tag{3}$$

By applying BGK model to Boltzmann equation [22], the equation for density distribution, f_i, is

$$f_i(\boldsymbol{r} + \boldsymbol{e}_i\Delta t, t+\Delta t) - f_i(\boldsymbol{r}, t) = \frac{1}{\tau_v}(f_i^{eq}(\boldsymbol{r}, t) - f_i(\boldsymbol{r}, t)) + F_i, \quad i = 1, 2, \ldots 9 \tag{4}$$

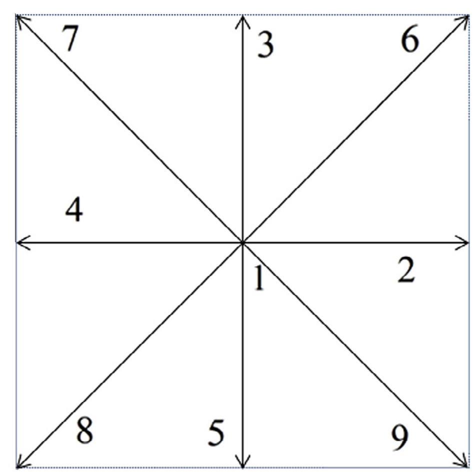

Fig. 1. Nine directions in D2Q9 model.

where Δt is the time step, and f^{eq} is the equilibrium distribution function:

$$f_i^{eq} = \rho\omega_i\left[1 + \frac{\boldsymbol{e}_i \cdot \boldsymbol{V}}{c_s^2} + \frac{(\boldsymbol{e}_i \cdot \boldsymbol{V})^2}{2c_s^4} - \frac{\boldsymbol{V} \cdot \boldsymbol{V}}{2c_s^2}\right] \tag{5}$$

where

$$\omega_i = \begin{cases} \frac{4}{9} & i=1 \\ \frac{1}{9} & i=2,3,4,5 \\ \frac{1}{36} & i=6,7,8,9 \end{cases} \tag{6}$$

Then the Navier–Stokes equation can be obtained through Chapman-Enskog expansion [16] when the kinematic viscosity ν is related to the relaxation time τ_v in Eq. (4) by:

$$\nu = c_s^2\left(\tau_v - \frac{1}{2}\right)\Delta t \tag{7}$$

where c_s is the speed of sound that is related to the lattice speed by $3c_s^2 = c^2$

The buoyancy force can be obtained as:

$$F_i = \Delta t \boldsymbol{G} \cdot \frac{(\boldsymbol{e}_i - \boldsymbol{V})}{p} f_i^{eq} \tag{8}$$

Download English Version:

https://daneshyari.com/en/article/657650

Download Persian Version:

https://daneshyari.com/article/657650

Daneshyari.com