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Analytical investigation of heat transfer enhancement in a channel partially filled with a porous material under local thermal non-equilibrium condition: Effects of different thermal boundary conditions at the porous-fluid interface

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ABSTRACT

Enhancement of forced convective heat transfer is analytically investigated in a channel partially filled with a porous medium under local thermal non-equilibrium (LTNE) condition. Thermally and hydrodynamically fully developed conditions are considered. The flow inside the porous material is modelled by the Darcy–Brinkman–Forchheimer equation. The thermal boundary conditions at the interface between the porous medium and the clear region are described by two different models. For each interface model exact solutions are developed for the solid and fluid temperature fields. The Nusselt number (*Nu*) associated with each interface model is derived in terms of the porous insert normalised thickness (*S*) and other pertinent parameters such as thermal conductivity ratio (*k*), Biot number (*Bi*), and Darcy number (*Da*). The differences between the two interface models in predicting the temperature fields of the solid and fluid phases and validity of the Local Thermal Equilibrium (LTE) assumption are examined. Subsequently, for each model the values of *S*, *Bi*, *k* and *Da* at which LTE holds are determined. Further, the maximum values of *S* up to that the two models predict LTE condition are found as a function of *Bi*, *k* and *Da*. For each model and for different pertinent parameters the optimum value of *S*, which maximises the *Nu* number, is then found. The results show that, in general, the obtained *Nu* numbers can be strongly dependent upon the applied interface model. For large values of *k* and *Bi*, there are significant disparities between the *Nu* numbers predicted by the two models. Nonetheless, for most values of *k* and *Bi*, and under different values of *Da* numbers both models predict similar trends of variation of *Nu* number versus *S*. The *Nu* number and pressure drop ratio are then used to determine the Heat Transfer Performance (HTP). It is found that for $S < 0.9$, HTP is independent of *Da* number and the model used at the porous-fluid interface. For $S > 0.9$, reduction of *Da* results in smaller values of HTP and signifies the difference between the values of HTP predicted by the two interface models.

1. Introduction

Energy saving is of primary importance in the design of heat exchangers. This has severely intensified the demand for high performance heat exchangers [1]. Significant efforts are therefore being made to improve the performance of heat exchangers. It is now well known that heat convection can be enhanced through using porous material [2]. Convective heat transfer in porous media features a wide range of engineering applications, including those in oil recovery, geothermal engineering, chemical reactors, hydrogeology, heat pipes, solid matrix heat exchangers and thermal insulation. In most of these applications, it is preferred not to fully fill the system with the porous medium. This is to avoid the significant pressure drops occurring in the fully filled systems. Therefore, partial filling is an attractive way of enhancing heat transfer, while maintaining the pumping expense at a reasonable level [3–5].

A number of authors have studied heat transfer performance of different partially porous filled systems. Poulikakos and Kazmierczak [6] studied fully developed forced heat convection in a channel partially filled with a porous medium. In this study, the porous material was attached to the channel wall. They found that there is an extremum porous thickness at which Nusselt number is

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Nomenclature

a_{sf}	interfacial area per unit volume of porous media (m^{-1})
A	constant parameter defined by Eq. (24)
B	constant parameter defined by Eq. (25)
Bi	Biot number, $\frac{a_{sf} h_{sf} h_0^2}{(1-\varepsilon)k_s}$
C	constant parameter defined by Eq. (27)
C_p	specific heat of the fluid, ($J\,kg^{-1}\,K^{-1}$)
Da	Darcy number, K/h_o^2
D_h	hydraulic diameter of the channel ($4\,h_0$)
f	friction factor
h_{sf}	fluid to solid heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
h_0	height of the channel (m)
h_p	porous substrate thickness (m)
K	permeability of the porous medium (m^2)
k	the ratio of solid effective thermal conductivity to that of the fluid, $(1-\varepsilon)k_s/(\varepsilon k_f)$
k_f	thermal conductivity of the fluid ($W\,m^{-1}\,K^{-1}$)
$k_{f,eff}$	effective thermal conductivity of the fluid, εk_f
k_s	thermal conductivity of the solid ($W\,m^{-1}\,K^{-1}$)
$k_{s,eff}$	effective thermal conductivity of the solid, $(1-\varepsilon)k_s$
Nu	Nusselt number
O_1	constant parameter defined by Eq. (51b)
O_2	constant parameter defined by Eq. (51c)
p	pressure (Pa)
q	heat flux ($W\,m^{-2}$)
Re	Reynolds number, $Re = \rho \bar{u} h_0/\mu$
S	ratio of the porous medium thickness to the channel height, h_p/h_0
$S_{crt,\Delta\Theta}$	critical S up to which the two models A and B show similar results of the maximum temperature difference between two phases
$S_{crt,LTE}$	critical value of S up to which the LTE condition validates
$S_{opt,Nu}$	optimum value of S which maximises the Nusselt number
$S_{crt,Nu}$	critical value of S up to which both models predict the same Nu number
T	temperature (K)
T_m	average temperature (K)
u	longitudinal velocity (m/s)
$\bar{u}$	average velocity
u_r	characteristic velocity, $-\frac{h_0^2}{\mu}\frac{\partial p}{\partial x}$
U	dimensionless velocity, u/u_r
$\bar{U}$	dimensionless average velocity
x	longitudinal coordinate (m)
y	transverse coordinate (m)
Y	dimensionless y coordinate, y/h_0
Z	constant parameter, $\sqrt{1/Da}$

Greek symbols

γ	ratio of wall heat flux to the heat flux at the interface, $q_w/q_{interface}$
Γ	constant parameter defined by Eq. (54e)
ε	porosity of the porous medium
Θ	dimensionless temperature
μ	viscosity ($kg\,m^{-1}\,s^1$)
ρ	density (kg/m^3)
ξ	constant parameter used in Eq. (43)
ψ	constant parameter defined by Eq. (54e)
$\varphi 1$	constant parameter defined by Eq. (54b)
$\varphi 2$	constant parameter defined by Eq. (54c)
$\varphi 3$	constant parameter defined by Eq. (54d)

Subscripts

eff	effective property
exchange	internal heat exchange
f	fluid
f1	fluid in the clear region
f2	fluid in the porous medium
in	inlet
m	mean
o	open channel without porous material
p	porous medium
s	solid
w	wall
interface	the interface between the porous medium and the clear region

Superscripts

–	mean value
′,″,‴,⁗	first, second, third, and forth derivatives with respect to Y

minimum. Another study showed that the thermal performance of a conventional concentric tube heat exchanger could be improved by inserting high thermal conductivity porous substrates [7]. The transient, developing, forced-convection flow in a concentric annuli partially filled with porous substrates was studied under two configurations [7]. These included the porous substrate attached either to the inner or outer cylinder [7]. In both cases, the boundary in contact with the porous substrate was exposed to a sudden change in its temperature while the other boundary was kept adiabatic [7]. It was found that comparing to the clear pipe, porous substrates may improve the Nusselt number by factor of twelve [7]. An investigation concentrated on the developing region of parallel-plate ducts, showed that when a porous substrate is attached to the inner wall of one plate, the Nusselt number is lower than that in a clear channel [8]. The highest Nusselt Number, under this condition, is achieved by the fully filled porous duct [8]. A study of forced convection in a pipe partially filled with porous substrates, with porous material inserted at the core of the pipe, revealed an order of magnitude increase in the Nusselt number compared to the clear pipe [4]. The numerical simulations [4] and experimental observations [9] showed that Nusselt number increases through partial filling, while the pressure drop is less than that of a conduit fully filled with a porous medium. Importantly, the configuration of the porous insert in the pipe can have a substantial effect on the rate of heat transfer [5]. It has been demonstrated numerically that if the porous material with low thermal conductivity is attached to the pipe inner wall, the Nusselt number is lower than that of the clear pipe [5]. However, under the same configuration and for the high values of thermal conductivity the obtained Nusselt number is always higher than its equivalent in the clear pipe [5]. The same study [5] reported that for different porous thermal conductivities inserting porous material at the core of the pipe results in the heat transfer rates much higher than those in the clear pipe [5].

It has been, recently, shown that inserting porous material in a channel could increase the Nusselt number up to 50% of that in a clear channel [10]. Further, in a partially filled parallel plate channel for the Darcy number of 10^{-3} the Nusselt number is maximum at the porous thickness ratio of 0.8 [10]. This ratio is defined as the thickness of the porous layer divided by the spacing between the