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Natural convection of non-Newtonian power-law fluids on a horizontal plate

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ABSTRACT

The problem of natural convective boundary layer flow of a non-Newtonian power-law fluid over an isothermal horizontal plate, which does not admit a similarity solution, has been solved numerically using a time-marching finite difference method. The analysis shows that the velocity, temperature and pressure inside the boundary layer depend on two parameters, the non-Newtonian power-law index (n) and the generalised Prandtl number (Pr^*). For $n > 1$ (dilatant fluids), the u-velocity profiles reveal that the maximum velocity attained increases but the thickness of the boundary layer decreases as the value of n is progressively increased above unity. For $n < 1$ (pseudoplastic fluids), the reverse occurs and the boundary layer thickness increases to a great extent while the maximum velocity is reduced as the value of n is progressively decreased below unity. The magnitude of the normal velocity component at the edge of the boundary layer is found to be smaller for dilatant fluids and larger for pseudoplastic fluids as compared to Newtonian fluids. It has been found that the dilatant fluids show improved heat transfer characteristics as compared to Newtonian and pseudoplastic fluids at the same generalised Prandtl number. The non-existence of self-similar solutions for non-Newtonian power-law fluids has been established, thus showing the utility of the numerical method developed to solve the system of partial differential equations.

1. Introduction

Natural convection flow is driven by buoyancy forces generated by density differences that can be caused by temperature gradients in the fluid. Natural convection is commonly encountered in processes like cooling of electronic equipments, nuclear reactors, solar devices, in polymer processing industries, food industries, etc. [1–4].

The present paper deals with natural convection of non-Newtonian fluids on horizontal surfaces. Natural convection from vertical plates has been explored extensively. In comparison, the number of studies on natural convection from horizontal surfaces is rather limited. In case of a heated vertical plate, as the hotter fluid moves up, colder fluid comes in from the surrounding, principally in the horizontal direction. In case of a heated horizontal plate facing upward, on the other hand, the buoyancy force gives rise to a pressure gradient perpendicular to the plate which in turn results in a pressure gradient along the plate. It is the latter that drives the natural convective flow. Thus there is a significant difference between the flow physics of natural convection on vertical and horizontal surfaces. Unlike the boundary layer that forms due to forced convection, the boundary layer on a horizontal plate due to natural convection is such that $\partial p/\partial y \neq 0$ and $\partial p/\partial x$ cannot be neglected inside the boundary layer (even when $\partial p_\infty/\partial x$ is zero). Several of such subtle physics of natural convection above a horizontal plate have been included in the theory formulated in this paper.

Having explained the distinguishing features of horizontal surfaces, we turn our attention to the other important feature of the present paper that is the fluid is non-Newtonian in nature. The study of heat transfer in non-Newtonian fluids has gained much importance due to a large number of industries (food processing, heat exchanger and reactor cooling, biochemical processes, etc.) dealing with these types of fluids [5–7]. The boundary layer flow of non-Newtonian fluids exhibits characters different from that of the conventional Newtonian fluids due to the non-linear variation of the shear stress with strain rate. There are several models to describe non-Newtonian fluid behaviour [8]. The power-law model [8] has been used widely to describe the flow of non-Newtonian fluids, in which the viscosity is assumed to vary as follows:

$$\mu = \mu_0 \left|\frac{\partial u}{\partial y}\right|^{n-1} \tag{1}$$

where n is the power-law index, constant for a particular fluid. Depending on the value of n, fluids are classified into three broad

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Nomenclature

c_f^*	reduced skin-friction co-efficient
c_p	specific heat capacity (J/kg K)
Ec	Eckert number
Gr	Grashof number
Gr^*	generalised Grashof number
g	acceleration due to gravity (m/s^2)
h	heat transfer coefficient (W/m^2 K)
k	thermal conductivity of the fluid (W/m K)
L	reference length scale (m)
Nu^*	reduced Nusselt number
n	non-Newtonian power-law index
Pr	Prandtl number
Pr^*	generalised Prandtl number
p	static pressure (Pa)
$\bar{p}$	dimensionless static pressure
$\bar{p}^*$	stretched variable for dimensionless pressure
q_w	wall heat flux (W/m^2)
Re	Reynolds number
Re^*	generalised Reynolds number
T	temperature of the fluid (K)
u,v	components of velocity along and normal to the plate respectively (m/s)
$\bar{u}, \bar{v}$	components of dimensionless velocity
u_0	scaling velocity (m/s)
$\hat{v}$	dimensionless normal velocity before scaling
x, y	coordinates along and normal to the plate (m)
$\bar{x}, \bar{y}$	dimensionless coordinates along and normal to the plate
$\bar{x}^*, \bar{y}^*$	stretched variables for dimensionless coordinates
$\hat{y}$	dimensionless normal coordinate before scaling
$\Delta\bar{y}_1$	distance of first grid point from the plate normal to it

Greek symbols

α	thermal diffusivity (m^2/s)
β	coefficient of volume expansion of the fluid (/K)
δ	boundary layer thickness (m)
ε	a positive number less than unity depicting the deviation from Newtonian behaviour
η	similarity variable
θ	dimensionless temperature
θ^*	stretched variable for dimensionless temperature
μ	dynamic viscosity (Pa s)
υ	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
τ_w	wall shear stress (Pa)
ψ	dimensionless stream function
ψ^*	stretched variable for dimensionless stream function

Subscripts

w	value of the parameter at the plate surface
∞	ambient condition
0	reference value or initial value

categories: pseudoplastic (shear-thinning fluids) – $n < 1$, Newtonian fluids – $n = 1$, and, dilatant (shear-thickening fluids) – $n > 1$.

The study of natural convection of a Newtonian fluid from a vertical plate with both constant surface temperature and constant wall heat flux is given by Burmeister [9]. Schlichting and Gersten [10] have presented a similarity solution for natural convection of a Newtonian fluid past a horizontal plate, which they referred to as "indirect natural convection" for the reasons explained in the second paragraph. According to them, the first similarity solution for isothermal, semi-infinite, horizontal plate was given by Stewartson [11] who studied the case of a fluid with $Pr = 0.7$. Rotem and Claassen [12] studied the problem of free convection over a semi-infinite horizontal plate for power-law variation in plate temperature and constant wall heat flux, and through experiments showed how the boundary layer breaks down into large-eddy instability some distance from the leading edge. Recently, a similarity solution for natural convection of a Newtonian fluid for complex boundary conditions has been given by Samanta and Guha [13].

Acrivos [14] was the first to study laminar natural convection of power-law fluids for several geometries. The experimental studies of Gentry and Wollersheim [15], on isothermal horizontal cylinders, were in good agreement with the theoretical predictions of Acrivos [14]. Experiments on the free convection of pseudoplastic fluids over horizontal wires were performed by Ng and Hartnett [16], where, unlike the previous studies, the diameter of the cylinder was comparable to the boundary layer thickness. Shenoy and Mashelkar [17] gave an extensive review on free convection in non-Newtonian fluids. Huang and Chen [18] gave local similarity solution for the natural convection of power-law fluids past a vertical plate. The natural convection of a shear-thinning power-law fluid ($n = 0.95$) past an isothermal vertical plate has been studied by Ghosh Moulic and Yao [19]. Chamkha et al. [20] studied the unsteady natural convection of power-law fluid past a vertical plate in a non-Darcian porous medium. Mixed convection heat transfer from a horizontal plate to power-law fluids was studied by Wang [21]. However, the natural convection of non-Newtonian power-law fluids over a horizontal plate has not been studied till the present.

The present work studies the laminar natural convection boundary layer flow of a non-Newtonian power-law fluid over a semi-infinite horizontal flat plate maintained at a constant temperature. The analysis reduces to that of a Newtonian fluid when n (power-law index) equals 1, thus demonstrating internal consistency of the solution approach. Natural convection in both shear-thinning as well as shear-thickening fluids has been analysed in the present paper.

2. Mathematical formulation

The x-axis is aligned along the plate from the leading edge while the y-axis is directed normal to the plate against the direction of gravity. The quiescent ambient fluid is maintained at a uniform temperature T_∞ and pressure p_∞. The boundary layer equations for a horizontal plate invoking the Boussinesq approximation are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{3}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + g\beta(T - T_\infty) \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{5}$$

where the viscosity is given by