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Wall conduction effects in laminar counterflow parallel-plate heat exchangers

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A B S T R A C T

Wall conduction effects in multilayered, counterflow, parallel-plate heat exchangers are analyzed theoretically and numerically. The analysis, carried out for constant property fluids, considers a hydrodynamically developed laminar flow and neglects axial conduction both in the fluids and in the plates. The temperature field is expanded as an infinite series in terms of a complete set of eigenfunctions associated with sets of both positive and negative eigenvalues. In addition to the exact solution, an approximate solution that retains only the first two terms in the eigenfunction expansion is considered. The approximate two-term solution, which still incorporates the effect of higher order modes through apparent temperature offsets introduced at the inlet/outlet sections, provides an accurate representation for the temperature field away from the thermal entrance regions, thereby enabling simplified expressions for the wall and bulk temperatures, local Nusselt numbers, and overall heat-transfer coefficient. As main outcome of the analysis, it is seen that increasing the wall thermal resistance lowers the absolute value of both positive and negative eigenvalues—thus reducing heat-exchanger effectiveness—and increases the Nusselt number of the fluid with lower heat-capacity flow rate bringing it closer to its theoretical value $140/17 = 8.2353$ corresponding to a constant heat flux boundary condition. Moreover, the proposed two-term solution is seen to reproduce with great accuracy the dependence of the outlet bulk temperatures with the wall thermal resistance. The asymptotic solution for nearly-balanced heat exchangers is also obtained, providing closed-form analytical expressions for this limiting case of practical interest.

1. Introduction

Parallel-plate heat exchangers are widely used in chemical, pharmaceutical, food processing, and many other industrial applications. More recently, they have also found application in a variety of emerging branches of thermal engineering. Thus, they are currently used in miniaturized reaction systems involving heterogeneously catalyzed gas-phase reactions [1], in thermoelectric generators that convert low-grade thermal energy into electrical power [2,3], and in thermoacoustic engines and refrigerators [4]. In addition, they are a key component of many cryogenic systems [5–7].

In recent years, there has also been a growing interest in the development of polymer heat exchangers, due particularly to their high resistance to fouling and corrosion. Specifically, the use of polymers offers substantial weight, volume, space, and maintenance cost savings in many applications over metallic heat exchangers [8]. Nevertheless, the low thermal conductivity of polymer materials typically results in a dominant wall heat transfer resistance, which imposes serious restrictions on the thermal design and operation of these devices [9–11].

Progress in the analysis of parallel-plate heat exchangers has been significant in the last decades due to their simple geometry and well established flow conditions [12]. In particular, the analysis of the steady-state laminar heat transfer between different streams coupled through compatibility conditions at the boundaries constitutes the so-called conjugated Graetz problem [13–15]. Under certain simplifying assumptions—constant property fluids and fully developed laminar flow—the problem becomes linear and is amenable to an elegant solution based on eigenfunction expansions, which in counterflow systems involves sets of positive and negative eigenfunctions associated with sets of positive and negative eigenvalues [15–17].

The aim of this paper is to generalize the recent work on laminar counterflow parallel-plate heat exchangers carried out by Vera and Liñan [18,19] so as to include the effect of a finite wall thermal resistance. The analysis, based on the seminal contributions by Nunge and Gill [16,17], uses symbolic algebra to write closed form analytical expressions for the eigenfunctions, leading to an exact analytical eigencondition that provides the eigenvalues

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Nomenclature

A	expansion coefficient
$\bar{A}$	$\mathcal{O}(1)$ expansion coefficient in the limit $\lvert\epsilon\rvert \ll 1$
$\bar{B}$	$\mathcal{O}(1)$ expansion coefficient in the limit $\lvert\epsilon\rvert \ll 1$
a_i	channel half-width of fluid i
C_n	expansion coefficient corresponding to the n-th eigenfunction
c_i	specific heat of fluid i
f_n	n-th eigenfunction in the limit $\kappa_w \to \infty$
g_n	n-th eigenfunction for finite κ_w
$G_{n,i}$	contribution of order ϵ^n to the 0-th eigenfunction in the limit $\lvert\epsilon\rvert \ll 1$
h_i	heat-transfer coefficient of fluid i
k	dimensionless parameter, $(a_1k_2)/(a_2k_1)$
k_i	thermal conductivity of fluid i
k_w	thermal conductivity of the wall
l_n	contribution of order ϵ^n to the eigenvalue λ_0 in the limit $\lvert\epsilon\rvert \ll 1$
L	length of the heat exchanger
M	Whittaker's function, $M_{\kappa,\mu}(z)$
m	dimensionless parameter, $(a_2\mathrm{Pe}_2)/(a_1\mathrm{Pe}_1)$
Nu_i	Nusselt number of fluid $i, h_i(4a_i)/k_i$
Pe_i	Peclet number of fluid $i, 2a_iV_i/\alpha_i$
Pr_i	Prandtl number of fluid $i, \nu_i/\alpha_i$
Re_i	Reynolds number of fluid $i, 2a_iV_i/\nu_i$
T	temperature
$\widetilde{U}$	dimensionless overall heat-transfer coefficient
u_i	longitudinal velocity of fluid i
V_i	average velocity of fluid i
W	Whittaker's function, $W_{\kappa,\mu}(z)$
$w(y_i)$	weight function, $(3/4)(1-y_i^2)$
X	longitudinal distance from the inlet of fluid 1
Y_i	transverse distance from channel i symmetry plane
y_i	dimensionless transverse coordinate, Y_i/a_i

Greek letters

α_i	thermal diffusivity of fluid $i, k_i/(\rho_i c_i)$
Δ_i	bulk temperature offset of fluid i at the inlet
δ_w	thickness of the wall
ϵ	small parameter, $1-(mk)^{-1}$
ε	heat exchanger effectiveness
Γ	Gamma function, $\Gamma(z)$
κ	first argument of Whittaker functions
κ_w	dimensionless parameter, $(a_1k_w)/(\delta_w k_1)$
$\hat{\kappa}$	lumped variable, $m^{1/3}k$
λ_n	n-th eigenvalue
μ	second argument of Whittaker functions
ν	dimensionless local heat-transfer rate, $\partial\theta_1/\partial y_1\rvert_{y_1=1}$
ν_i	kinematic viscosity of fluid i
ρ_i	density of fluid i
θ_i	dimensionless temperature of fluid i
ξ	dimensionless longitudinal coordinate
$\Omega_n^{\pm}$	coefficients defined by Eq. (41)

Subscripts

i	subscript used indistinctly for fluids 1 and 2
in	inlet
L	length of the heat exchanger
m	bulk, or mixing-cup, temperature
n	corresponding to the n-th eigenvalue/eigenfunction
out	outlet
w	heat-exchanging wall

Superscripts

(0)	zeroth-order two-term solution
(1)	first-order two-term solution
$[N]$	$2(N+1)$-term truncated exact solution

numerically. In addition to the exact solution—expanded as an infinite series using the complete set of eigenfunctions—an approximate solution that retains only the first two modes in the eigenfunction expansion is derived. This approach, which still incorporates the effect of the higher order eigenfunctions through apparent temperature offsets induced at the inlet/outlet sections, provides accurate representations for the temperature field away from the thermal entrance regions. The accompanying expressions for the wall and bulk temperatures, local heat-transfer rate, overall heat-transfer coefficient, Nusselt numbers, and outlet bulk temperatures may be useful for engineering applications even for moderately short heat exchangers.

The paper is organized as follows. In Section 2, we present the dimensionless formulation of the problem, introducing the governing dimensionless parameters. In Section 3, we expand the solution as an infinite series of eigenfunctions and derive the linear system for the expansion coefficients. In Section 4, we propose an approximate two-term solution that involves only the lowest order eigenvalue and eigenfunction. In Section 5, we comment on the relevance of our results in the context of classical heat exchanger analysis. In Section 6, we validate the exact and approximate solutions against numerical solutions, and discuss the main effects of the wall thermal resistance on heat-exchanger performance. Finally, in Section 7, the main conclusions of the analysis are presented.

As additional material, to be used in the discussion of results, in Appendix A we give the asymptotic solution in the limiting case of nearly-balanced heat exchangers, while in Appendix B we address the degenerate case of highly-unbalanced heat exchangers.

2. Problem formulation

In this paper we analyze the heat transfer between two constant-property newtonian fluids flowing through a multilayered counterflow parallel-plate heat exchanger composed by a relatively large number of channels separated by plates of finite thickness, δ_w, and thermal conductivity, k_w. The conducting plates allow the exchange of heat through a section of length L, presenting insulated regions at both ends of the heat exchange region, where no heat transfer is allowed [20]. In the configuration considered here, the two fluids, denoted by 1 and 2 (hereafter, the subscript i will be used indistinctly for both fluids, $i = 1, 2$), flow in opposite directions in adjacent channels. Then, if the characteristic cross-sectional dimension of the heat exchanger is large compared with the channel width, $2a_i$, the temperature field, as seen with this scale, appears as periodic in the transverse direction, with period $2(a_1 + a_2)$. As a result, when describing the temperature field in the unitary cell of the heat exchanger we can use symmetry boundary conditions at the channel symmetry planes, where no thermal energy can be transferred.

Fig. 1 presents a sketch of the theoretical model under study, including the coordinate system, the velocity profiles, the inlet