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Thermally hiding an object inside a cloak with feeling

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ABSTRACT

A thermal cloak can hide an object inside the cloak from the detection by measuring external heat flow (outside the cloak); this behavior is called the cloaking effect. However, such a thermal cloak has a limitation. Namely, the object is always located inside the cloak's central region with a homogeneous temperature distribution, and hence this object cannot feel the flow of heat. To overcome the limitation, we develop a coordinate transformation method to design a complementary material, and add it into the central region of the cloak. As a result of finite element simulations, in case of external heat flow, the temperature distribution in this central region becomes inhomogeneous and hence the object feels the flow of heat indeed. Meanwhile, the cloaking effect remains the same, or almost the same as the cloak without such a complementary material inside. Thus, it becomes possible to use complementary materials to design thermal devices where heat conduction can be controlled at will.

1. Introduction

With an attempt to control heat conduction at will, Fan et al. [1] designed thermal cloak by utilizing coordinate transformation method as early as 2008: They started from the form invariance of the thermal conduction equation in distorted coordinates, and proposed a type of thermal cloak for steady state heat flow (i.e., the temperature, T, is not a function of time) by tailoring anisotropic and inhomogeneous materials appropriately. The thermal cloak can hide an object inside the cloak from the detection by measuring the external heat flow (which originates from a temperature gradient outside the cloak). This behavior is called the cloaking effect; it is because the cloak and object do not affect the distribution of temperature outside the cloak as if they are absent. In 2012, Narayana and Sato [2] experimentally realized this cloaking effect by using forty alternating layers of two materials, one (or the other) of which has a high (or low) thermal conductivity. In the mean time, while some theoretical scholars proposed to achieve such thermal cloaks by utilizing simplified methods based on homogeneous materials [3], other theoretical [4] or experimental [2,5,14] scholars extended the thermal cloaks from steady state heat flow to unsteady state heat flow (for which T changes as time varies).

However, all the thermal cloaks reported in the literature [1–12,15] have a common limitation: the cloaked object cannot feel the flow of heat because it is located inside the cloak's central region where the temperature distribution is always homogeneous no matter whether external heat flow exists or not. In other words, the object hidden inside the cloak has no feeling for the external heat flow (if any), or equivalently it is "blind" to the existence of external heat flow. Clearly, if the limitation is overcome in thermal cloaks, the cloaking effect could be more practicable, especially for the cloaked object. We understand that conventional thermal cloaks have two mainly properties. One is thermal invisibility (namely, the temperature distribution pattern outside a cloaking region does not change no matter whether there is a cloak or not), and the other is thermal shielding (i.e., preventing the flow of heat into the central area of a cloak). Since thermal shielding can be achieved by many other methods (say, by using heat-insulating materials), thermal invisibility is the unique feature of thermal cloaks. Fortunately, the feeling cloak designed in this work still has the same feature of thermal invisibility. In the meantime, giving up the property of thermal shielding makes the object located within the central area of the cloak feel changes of a temperature gradient, and thus this object has feelings. In other words, during the process of heat conduction, our design could be a good choice under the following condition: a person (or an object), who lives in the central area of the cloak, wants to know what happens outside while he still hopes to prevent himself from being detected by others by measuring the change of temperature gradient pattern outside the cloaking area. That is, the cloaked object itself would no longer be "blind" to the presence of external heat flow. In a recent paper [13], Gao and Huang managed to overcome the limitation by

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using complementary materials [17,16] to cloak an object that is located outside the cloak. As a result, the object can feel the external heat flow, but at a cost of specific design of configurations [13]: the cloak's parameters must be changed according to geometrical or material properties of the object. In other words, according to the design proposed in Ref. [13], *different cloaks must be used to cloak different objects*. In order to reduce the complexity of this kind of design, here we raise a question: can one let the object, which stays inside the central region of the cloak, feel the flow of heat? If so, the complexity of the former design in Ref. [13] could be avoided, which is because *one cloak can then be used to cloak different objects*. For this purpose, we shall propose a recipe according to coordinate transformation method [1,12].

2. Recipe based on coordinate transformation method

Without loss of generality, we omit the heat source, and start from the case of steady state heat flow in a two-dimensional thermal cloak with coordinates (r, θ); see Fig. 1. Here r denotes the radial coordinate, and θ represents the angular coordinate. In this case, the heat diffusion equation satisfies $\nabla\tilde{\kappa}\nabla \cdot T = 0$, where $\tilde{\kappa}$ denotes a thermal conductivity. When we compress the annulus region with $r_0 < r < r_b$ to the region with $r_a < r < r_b$, the transformation method can be expressed as

$$\frac{r_b - r_a}{r_b - r_0} r + l_1 = r', \tag{1}$$

where $l_1 = r_b(r_a - r_0)/(r_b - r_0)$ and r' denotes the radial coordinate in the distorted coordinates. It is worth noting that setting $r_0 \to 0$ in Eq. (1) reduces to the transformation for constructing the original thermal cloak reported in Refs. [1,12]. This transformation method (Eq. (1)) leads to the cloaking parameters (namely, thermal conductivities) in the distorted coordinates; they can be derived from the following relation between $\tilde{\kappa}'$ (thermal conductivity in the distorted coordinates) and $\tilde{\kappa}$ (thermal conductivity in the original or regular coordinates),

$$\tilde{\kappa}' = \frac{\mathbf{J}\tilde{\kappa}\mathbf{J}^{t}}{\det(\mathbf{J})}, \tag{2}$$

where $\mathbf{J}$ is the Jacobian transformation matrix between the distorted and original coordinates, $\mathbf{J}^t$ is the transposed matrix of $\mathbf{J}$, and $\det(\mathbf{J})$ is the determinant of $\mathbf{J}$. Then, the cloaking parameters (namely, the radial and angular thermal conductivity, $\tilde{\kappa}_{r,cloak}$ and $\tilde{\kappa}_{\theta,cloak}$) can be figured out as

$$\begin{aligned} \tilde{\kappa}_{r,cloak} &= \kappa_{r,cloak}\kappa_0 = \frac{r' - l_1}{r'}\kappa_0, \\ \tilde{\kappa}_{\theta,cloak} &= \kappa_{\theta,cloak}\kappa_0 = \frac{r'}{r' - l_1}\kappa_0, \end{aligned} \tag{3}$$

where κ_0 is the thermal conductivity of background. Fig. 2(a) shows how $\kappa_{r,cloak}$ and $\kappa_{\theta,cloak}$ vary with r. Here we should remark that l_1 is important to achieve the cloaking effect. When $r_0 = 0$, $l_1 = r_a$, and then heat flux in the radial direction vanishes. This facts means that due to continuity, the flux in the central region with $r < r_a$ also vanishes. In other words, there is a homogeneous temperature distribution inside the central region ($r < r_a$). On the other hand, heat flux in the angular direction turns to be infinite. This allows heat flux detouring the central region ($r < r_a$). Moreover, symmetry of the device ensures the heat flux to propagate forward as if there is nothing (except for the background with a thermal conductivity, κ_0) in the space.

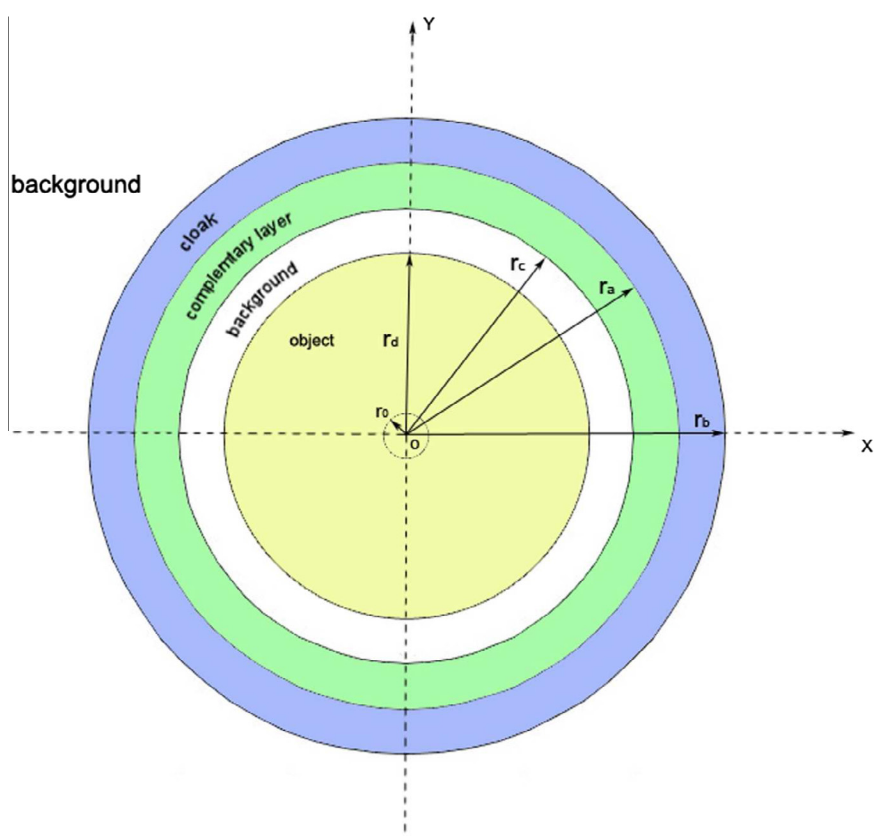

Fig. 1. Schematic graph showing a cloak and a complementary layer: the cloak is located within the region of radius r satisfying $r_a < r < r_b$, and the complementary layer is situated within $r_c < r < r_a$. X and Y axes are also added. Other details are indicated in the main text.