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Mixed convective heat transfer from a heated sphere at an arbitrary incident flow angle in laminar flows

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ABSTRACT

The mixed convection from a heated sphere for an arbitrary flow incident angle (θ) at low to moderate Reynolds numbers ($1 \leqslant Re \leqslant 100$) and Richardson numbers ($0 \leqslant Ri \leqslant 5$) is studied by an immersed boundary method, thereby investigating the influence of different flow incident angles ($0 \leqslant \theta \leqslant 180^\circ$) on the buoyancy flow and heat transfer. The numerical method is validated by comparing the results with the simulation results of pure forced convection as well as those of mixed convection with assisting flow (0° flow incident angle) published in the literature. Extensive simulations for a wide range of different incident flow angles have been performed. New correlations are obtained for the overall Nusselt number (Nu) in terms of θ, Ri, and Re, showing a quadratic decrease in Nu with respect to θ only for $0^\circ \leqslant \theta \leqslant 90^\circ$ (aiding and cross flow) and a half bell-shaped decrease in Nu for $90^\circ < \theta \leqslant 180^\circ$ (opposed flow). The combined treatment of mixed convection for the completely upward flow, cross flow, and completely downward flow (i.e. at incident angle 0°, 90° and 180°, respectively) was achieved showing almost linear relationships between the heat transfer rates and Ri for $Ri \geqslant 1$.

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1. Introduction

Heat transfer on a heated sphere immersed in a viscous fluid can be treated as a free convection or a forced convection problem. However, in most heating and cooling applications in real world, the convective heat transfer is of a mixed type. Mixed convection occurs whenever there is a forced fluid flow along with fluid motion driven by a temperature difference between the body and the surrounding fluid. The free convection occurs along the direction of gravity and the forced convection depends on the direction of the forced flow. The two directions are not necessarily the same. The overall heat transfer of a sphere resulting from a mixed flow is significantly influenced by the direction of the forced flow.

The Richardson number (Ri) characterizes the importance of free convection with respect to forced convection. Yuge [1] found that the forced convection is predominant when $Ri < 1$ and the free convective heat contribution becomes negligible when $Ri < 0.01$. This was confirmed by Ziskind et al. [2] who noticed that when $Ri > 1$, the buoyancy induced flow dominates over the forced convective flow. A pure forced convection therefore occurs at $Ri = 0$ while a pure free convection occurs at $Ri = \infty$. $Ri = 1$ is the intermediary that separates strong free convection from strong forced convection.

Most researchers have approached the problem of mixed convection by dividing it into two regimes based on the relationship between the imposed flow and buoyancy flow directions [3–5]: aiding and cross flow ($0 \leqslant \theta \leqslant 90^\circ$) and opposing flow ($90^\circ < \theta \leqslant 180^\circ$). Here, θ is the angle between the direction of the forced flow and the direction of the free convection. The first theoretical study on mixed convection for aiding flow past a sphere was done by Acrivos [6] who used a laminar boundary layer approximation for large Re. Hieber and Gebhart [7] used a matched asymptotic expansion to study the cases of small Gr and Re. Yuge [1] experimentally investigated the heat transfer between a gas and a spherical surface, defining the forced convection Nusselt number (N_R) and natural convection Nusselt numbers (N_G) for $3.5 \leqslant Re \leqslant 110$ and $0 \leqslant Gr \leqslant 1818$ in the aiding, cross, and opposing mixed flow regimes. Klyachko [8] critically analyzed Yuge's study and refined the correlations for the mixed convection of a sphere in air by identifying a critical Richardson number Ri_{crit} when either the inertial forces or buoyancy forces prevail in the heat transfer process. An alternative correlation for Yuge's experimental results has been provided by Armaly et al. [9] in terms of N_R and N_G. Chen and Mucoglu [10] numerically investigated the assisting and opposing flows past a sphere using a finite difference method and studied the surface heat transfer on a sphere with constant surface

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Nomenclature

A	radius of a sphere
Bi	Biot number
C	specific heat
D	diameter of a sphere
f	force density
F	surface boundary force
Gr	Grashof number
H	grid spacing
K	thermal conductivity
L,l	length variable
Nu	overall Nusselt number
p	pressure
Pe	Peclet number
Pr	Prandtl number
Ra	Rayleigh number (*GrPr*)
Ri	Richardson number
Re	Reynolds number
s	surface coordinate
T	temperature
t	time variable
u, U	velocity
x,X	position coordinate

Greek symbols

ρ	density
λ	energy density function
Ω	entire domain
β	thermal expansion coefficient
μ	dynamic viscosity
Θ	dimensionless temperature
θ	flow angle of incidence
Λ	energy flux at boundary

Subscripts/superscripts

0	value of ambient fluid
f	property related to fluid
s	property related to sphere
G	natural convection
R	forced convection

temperature and uniform heat flux for large *Gr* and *Re*. They found the dynamics of mixed convection in the configuration of opposing flow to be more complex than that of assisting flow.

Buoyancy effects in mixed convection have also been extensively studied. Chen and Mucoglu [5] concluded that the local Nusselt number increases with increasing buoyancy force for aiding flow, and decreases with increasing buoyancy force for opposing flow. This finding was also confirmed by Wong et al. [11] and Gopmandal et al.[12]; the latter studied thermal buoyancy flows around a sphere translating in a quiescent fluid and found that the separation point shifted towards upstream for the buoyancy-opposed flow ($Ri < 0$), resulting in a larger recirculation downstream.

Since the Nusselt numbers for either forced or natural convection can be predicted experimentally, several authors have combined these results to predict the *Nu* for mixed convection. Tang et al. [3] explored the effective diameter scalar addition method which was first developed by Kirk and Johnson [4] to predict mixed convection in the cross flow and aiding flow regions. The former used a composite power curve to predict opposing flow mixed convection. Kirk and Johnson [4] conducted experiments on mixed convection heat loss from spheres in air for *Re* varying from 123 to 1070 and collected data for cross flow, aiding flow, and opposing flow with Grashof number values in the order of 10^5. Their results indicated that cross and aiding flow ($0 \leqslant \theta \leqslant 90^\circ$) must be treated separately from opposing flow ($90^\circ < \theta \leqslant 180^\circ$). They also found that over a wide range of *Re*, opposing flows show a maximum and minimum *Nu*, which confirms the results of Yuge [1].

From previous studies on the subject matter, a generalized correlation equation for mixed convective flow over a sphere for all flow regimes and a wide range of flow numbers has not been achieved. A clear understanding of the heat transfer characteristics for mixed flow at an arbitrary angle is not available. Churchill [13] obtained a correlation equation for mixed convective flow about a sphere which is applicable only to aiding flow; he also stated that attempts to generalize the correlating equation in some simple fashion to include forced convection oblique to or opposed to free convection were unsuccessful. The only existing correlation of the heat transfer rate in terms of the incident flow angle was obtained by Tsubouchi and Sato [14] for $\theta \geqslant 45$, $3 \times 10^{-2} \leqslant Gr \leqslant 40$, $0.3 \leqslant Re \leqslant 30$, and $Ri > 3/4$ in air as follows:

$$Nu = 2 + 0.5[Re^2 + (4/3)Gr - (4/\sqrt{3})ReGr^{1/2}\cos(\theta)]^{1/4} \qquad (1)$$

In the present study, a three dimensional immersed boundary based method has been developed to solve the mixed convection from a heated sphere for an arbitrary flow incident angle at low to moderate Reynolds numbers $1 \leqslant Re \leqslant 100$ and Richardson numbers $0 \leqslant Ri \leqslant 5$. The influence of different flow incident angles on the buoyancy flow and heat transfer rate is investigated numerically. We first apply the numerical method to study the forced convection as well as the mixed convection with assisting flow. The simulation results are compared to those published in the literature and good agreements are found. We then extend the method to study the mixed convection at various flow incident angles. The flow structures are analyzed. Based on the present simulation results, a new correlation is derived for the Nusselt number in terms of a wider range of flow numbers and incident angles.

2. Problem description and the immersed boundary method (IBM)

The immersed boundary method (IBM) [15] combined with a direct forcing scheme [16] has been implemented for solving particle motions and fluid velocity fields of particulate flows by Feng and Michaelides [17]. It has also been extended to solve heat transfer of moving particles in a viscous fluid by Feng and Michaelides [18,19] and by Feng [20]. For the present problem, the sphere is considered fixed with no rotational or translational velocity. Two types of grids are used to solve the solid interactions: one is a fixed regular background grid or Eurlian grid for the entire flow domain, and the other is a Lagrangian grid for outlining the immersed solid boundary of the particle.

For the momentum interaction between the fluid and the solid boundary, the presence of the solid boundary can be represented by a virtual boundary with a distribution surface force $\vec{F}$ assigned on this boundary or volumetric force density $\vec{f}$ around the boundary region. The virtual boundary behaves the same as the solid boundary: when fluid approaches the virtual boundary, a distributing force arises that repels the fluid and prevents it from penetrating; this distribution force depends on the incoming flows and can be conveniently computed by a direct forcing scheme. The force density function enforces the no-slip boundary condition on the particle surface. Similarly, with respect to thermal interaction between the fluid and the solid boundary, a thermal energy density