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A topological optimization procedure applied to multiple region problems with embedded sources

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ABSTRACT

The main objective of this work is the application of the topological optimization procedure to heat transfer problems considering multiple materials. The topological derivative (D_T) is employed for evaluating the domain sensitivity when perturbed by inserting a small inclusion. Electronic components such as printed circuit boards (PCBs) are an important area for the application of topological optimization. Generally, geometrical optimization involving heat transfer in PCBs considers only isotropic behavior and/or a single material. Multiple domains with anisotropic characteristics take an important role on many industrial products, for instance when considering PCBs which are often connected to other components of different materials. In this sense, a methodology for solving topological optimization problems considering anisotropy and multiple regions with embedded heat sources is developed in this paper. A direct boundary element method (BEM) is employed for solving the proposed numerical problem.

1. Introduction

Shape and topology optimization are of high importance for engineering problems. The search for optimization methods with low computational cost, which deliver the best solutions for the problem under consideration, play an important role in the academic community. The choice of the numerical method to be employed is based on a number of features. Among the numerical methods that can be classified as classical, and which have been successfully employed for topology optimization, are the SIMP (solid isotropic method with penalization), ESO (evolutionary structural optimization) and level set methods. All these methods are generally performed by taking the finite element method (FEM) as the numerical method of choice, and are often applied for solving elasticity problems. According to Shiah et al. [1], materials with anisotropic properties have been employed for a great number of applications since the earlier 1960s. High performance is achieved when a composite is constructed by combining two or more materials. Despite its importance, very few works are found in the literature considering topology optimization for anisotropic materials applied for heat transfer, when compared to elasticity problems. Li et al. [2] developed a computational procedure based on FEM and ESO for the topology design of heat conduction in isotropic fields. Zhang and Liu [3] developed a new method based on topology optimization for solving heat conduction problems for isotropic media with distributed heat sources. This method was able to reconstruct the conducting paths by distributing high conductive materials. Heat flow control within a composite material was studied by [4]. A gradient based optimization routine coupled with an FEM solver was employed in an iterative process for determining the distribution of the thermophysical parameters. Based on these parameters, the optimal conductivity heat transfer path in the composite was designed. Another concept recently employed for topology optimization using BEM instead of FEM is the topological derivative [5,6]. Some results employing D_T were also performed using FEM, but it is worth noting that the BEM characteristics are attractive for optimization procedures once this method has no mesh dependence and low computational cost. The D_T measures the sensitivity of a given shape functional when the domain is perturbed by an infinitesimal perturbation, such as the insertion of holes, inclusions and even cracks. This concept has been successfully used for a wide range of problems in addition to topology optimization, such as image processing [7] and fracture mechanics [8]. Many efforts for extending this concept for more complex problems have been done in the last years. Recently, [9,10] obtained a closed form for the D_T considering the total potential energy associated to an anisotropic

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and heterogeneous heat diffusion problem, when a small circular inclusion of the same nature of the bulk phase is introduced at an arbitrary point of the domain. This closed formula was derived in its general form and particularized for heterogeneous and isotropic media, showing agreement with the Amstutz [11] derivation. On the basis of previous relevant work by one of the authors [5,12], this paper aims at extending an optimization procedure for heat transfer problems considering heterogeneous materials using BEM and D_T. The derivation of an integral equation valid over all the external boundaries of the different material regions, without due consideration for the interfaces between them, is not impossible, despite the complexity involved. Alternatively, a conforming mapping technique is implemented to reduce the steady-state anisotropic field to an equivalent isotropic domain, avoiding some new derivations [1]. The influence of the anisotropic conductivity properties imposed to the inclusion and matrix on the final topology will be investigated. The manuscript is organized as follows. The BEM treatment of multi-domain mapping is described in Section 2. In Section 3, the architecture of the optimization process is presented, as well as the D_T sensitivity formula used. A sample of two-dimensional numerical problems is outlined. The numerical results are presented and discussed in Section 5. Finally, the conclusions are given in Section 5.

2. BEM treatment of multi-domain mapping

Many research works have devoted substantial efforts on developing efficient and robust topology optimization procedures during the last decades. The present work is focused on anisotropic multi-regions for heat diffusion problems. When dealing with non-homogeneous composites, it is usually necessary to split the domain into several different materials held together, and treat each one in turn. The derivation of an integral equation valid over all the external boundaries of the different material regions without due consideration for the interfaces between them is not impossible, despite the complexity involved. Alternatively, a conforming mapping technique can be used to reduce the steady-state anisotropic field to an isotropic equivalent domain, avoiding some new derivations. Some works have successfully employed a linear coordinate transformation for solving anisotropic thermal field problems with FEM and/or BEM. The first works devoted to the mapping technique were presented by [13,14]. Recently, Shiah and Tan [15] presented a method for reducing a three-dimensional steady-state anisotropic field problem to an equivalent isotropic one, governed by the Laplace equation in a mapped domain. Another work presented by [1] expanded this technique to the heat conduction in composites consisting of multiple anisotropic media for 2D and 3D. Despite the mapping technique formulation being available for 3D problems, only 2D problems have been considered. Furthermore, the D_T formula for calculating the domain sensitivities due to a perturbation caused by the insertion of a small inclusion is only valid for 2D problems [9]. Problems governed by the Poison equation are well known in the BEM literature [16]. The boundary integral equation for this problem is as follows:

$$c(\eta)\phi(\eta) = \int_{\Gamma} q(\eta,\xi)U(\eta,\xi)d\bar{\Gamma}(\xi)\cdots - \int_{\Gamma}\phi(\xi)U(\eta,\xi)d\bar{\Gamma}(\xi) - \sum_{m=1}^{n}\overline{b_m}U(\eta,M_n) \quad (1)$$

where ϕ and q represent the temperature and its normal gradient, respectively. The variable n represents the unit outward normal vector along the boundary while $\bar{\Gamma}$ is used to denote the boundary on the mapped domain.

The source and field points are designated by the variables η and ξ. The value of $c(\eta)$ depends on the geometry at η. The variable $\overline{b_m}$ is the mth internal heat-source point. The fundamental solutions for the potential and its gradient are represented by $U(\eta,\xi)$ and $T(\eta,\xi)$ and given by,

$$\begin{aligned} &U(\eta,\xi)=\tfrac{1}{4\pi r} \quad \text{and} \quad T(\eta,\xi)=-\tfrac{1}{4\pi r^2}n_i r_{,i} \quad \text{for} \quad 3D\\ &U(\xi)=\tfrac{1}{2\pi}\ln(\tfrac{1}{r}) \quad \text{and} \quad T(\xi)=-\tfrac{1}{2\pi r}n_i r_{,i} \quad \text{for} \quad 2D \end{aligned} \quad (2)$$

After the usual discretization of the boundary into boundary elements, Eq. (1) is applied at each boundary nodal point (η), generating the system of equations for a single domain. The original geometry (Γ) is mapped into an isotropic equivalent domain ($\bar{\Gamma}$) by using a linear coordinate transformation,

$$\begin{aligned} &[X_1X_2] = [F(K_{ij})][x_1x_2]^T\\ &[x_1,x_2] = [F^{-1}(K_{ij})][X_1X_2]^T \end{aligned}$$

where K_{ij} is the conductivity coefficient and $[F(K_{ij})]$ as well as $[F^{-1}(K_{ij})]$ are defined by,

$$F=\begin{bmatrix}\sqrt{\Delta}/K_{11} & 0\\ -K_{12}/K_{11} & 1\end{bmatrix} \quad F^{-1}=\begin{bmatrix}K_{11}/\sqrt{\Delta} & 0\\ -K_{12}/\sqrt{\Delta} & 1\end{bmatrix} \quad \Delta = K_{11}K_{22}-K_{12}^2$$

Now, the collocation process for solving the integral equation (1) should be done on the distorted domain and this procedure also requires a mapping of the Neumann boundary conditions, as depicted in Fig. 1. It is important to note that the nodal potentials remain unchanged for corresponding points between the physical (x_1,x_2) and mapped coordinate systems (X_1,X_2), since the temperature is a scalar field.

The Neumann boundary conditions are mapped according to the relation,

$$\frac{dT}{d\bar{n}} = \left(\frac{\partial T}{\partial x_1}\frac{K_{xx}}{\sqrt{\Delta}} + \frac{\partial T}{\partial x_2}\frac{K_{xy}}{\sqrt{\Delta}}\right)\bar{n}_1 + \left(\frac{\partial T}{\partial x_2}\right)\bar{n}_2 \quad (3)$$

Taking into account the boundary conditions imposed to the problem, a set of simultaneous equations for unknown and known temperature or its normal derivative at nodal points may now be solved by standard methods. Once the system of equations is solved, an inverse mapping must be further employed for recovering the potential gradients on the original domain, as follows,

$$\frac{dT}{dn} = \bar{\varphi}_i^T F \frac{F^T\bar{n}^T}{\left|F^T\bar{n}\right|} \quad (4)$$

where $\bar{n}$ is the outward normal vector on the mapped domain. The variables $\bar{\varphi}_i$ represent the temperature gradients along the mapped boundary, and are defined as,

$$\bar{\varphi}_i^T = \begin{pmatrix}\frac{\partial T}{\partial X_1} & \frac{\partial T}{\partial X_2}\end{pmatrix} \quad (5)$$

with each temperature gradient of Eq. (5) calculated by

$$\begin{aligned} &\partial T/\partial x_1 = (dT/d\bar{n})\bar{n}_1\\ &\partial T/\partial x_2 = (dT/d\bar{n})\bar{n}_2 \end{aligned} \quad (6)$$

When dealing with non-homogeneous media, appropriate thermal compatibility and equilibrium conditions along the interfaces of conjoint materials must be supplied. As explained before, when the anisotropic domain is transformed to an equivalent isotropic one, it results in a deformed geometry. The inserting of an inclusion with different properties inside the matrix will result in an overlap or separation of the interfaces of the conjoint materials (see Fig. 1). For a non-cracked interface between isotropic materials, the compatibility equation is imposed as

$$T^1 = T^2 \quad (7)$$

where the superscripts denote materials 1 and 2, respectively. The above relation still holds for anisotropic materials, but special