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The emergence of the constructal element in tree-shaped flow organization

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ABSTRACT

This paper presents how the elemental systems of constructal tree-shaped flows originate (Bejan, 2000) [1]. It is at the elemental system level where the transport (flow) turns from slow (volumetrically) to fast (aligned) thus making the essential connection between an infinity number of points to a finite entity. That is the only mathematically and physical way one can make such connection. The elemental system is the smallest scale of volume-to-point flows that occur in Nature (e.g., lungs, river deltas, botanical trees and their roots, canopies and leaves veins) and Engineering (e.g., photovoltaic metallic contacts networks). They are explained and predicted by Constructal Theory (CT). In this paper the elemental system starts its design evolution in a simpler configuration to reach the configuration that is otherwise assumed in Constructal Theory and fractals publications alike. The design evolution takes place in a combination of evolutionary computing, Constructal Law and finite element analysis. An original computer code performs the algorithm. Results showed that (a) the constructal element evolves to a configuration very similar to ones customarily assumed in CT, but with the high-conductivity insert shorter, leaving a gap at the far end; (b) the resulting configurations are nearly path independent; and (c) the gain in performance resembles an S curve, and it is explained theoretically.

1. Introduction

There are countless examples of volume-to-point types of flow in Nature and Engineering (e.g., lungs, river deltas, botanical trees and their roots, canopies and leaves veins, photovoltaic metallic contacts networks, e.g., [1]). Those systems are finite in size and perform a specific function that requires some quantity to flow through them. In almost all the times they have at least two distinct parts: one that performs the main function and where the flow is volumetrically and another part that feeds the whole with a faster flow (e.g., lungs).

A feasible configuration to connect an infinity number of points to a single point (or finite area) in functioning systems cannot be achieved by an infinite number of channels that goes down to infinitesimal sizes. Otherwise, the channels would take the entire volume leaving no space for functioning parts. The existing configurations that does the task and still provide easy flow have dendro-like patterns. Those tree-networks are finite in size and they have recognizable and measurable features [2–6].

The topic has been reviewed in Refs. [7,8]. A great distinction has been set between the approach by fractals [9–11] and by the Constructal Theory [12,13]. While the former successfully produces images with great resemblance of natural systems it does not rely on physics. The latter, on the other hand, claims that forms, patterns and configurations can be determined and explained by physics. By the constructal approach it was possible to determine tree-like configurations that solved the volume-to-point flow problem [8,12–20].

Constructal Theory has been recently reviewed in Refs. [7,13,20]. It is based on the Constructal Law, that states "For a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that provides greater and greater access to the currents that flow through it". Literature [7] now shows Contructal Law provides explanations for why a broader variety of designs, configurations, structures, patterns and rithms occurred in the natural and in the anthropic realms alike. It sets the basis to understand why sometimes quantities are minimized and sometimes quantities are maximized to determine designs. Ultimately, the Constructal Theory explains why the emergence of configurations must be a physics phenomenon and treated as such.

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Nomenclature

A	fixed area of whole elemental system, m^2
A_p	area of the high conductivity insert or patch, m^2
a, b	curve fit coefficients
D	width of the outlet of the system, m
H_0	height of the elemental system, m
h_x, h_y	height (y) and width (x) of the high conductivity tissue, m
k, k_p	thermal conductivity of the original tissue and the defective patch, W/m K
L_0	length of the elemental system, m
g	current generation
G	number of generations
P	progeny size, [individuals]
$\dot{Q}$	heat flow, W
$\dot{q}''$	heat generation, W/m^2
$\dot{q}'''$	volumetric heat generation, W/m^3
T_0	outlet patch temperature, K
T_{max}	maximum temperature in the system, K
$T(x,y)$	temperature field, K
x, y	coordinates, m
x_c, y_c	coordinates of the center of the high conductivity patch, m

Greek symbols

ΔT_{max}	maximum difference of temperature, K
Θ	dimensionless temperature field within the system
Θ_{max}	maximum dimensionless temperature within the system
$\Theta_{max(0)}$	maximum dimensionless temperature within the system at the first generation
φ	porosity of the system with the high conductivity patch, $\varphi = \tilde{h}_x\tilde{h}_y$
γ	geometric progression rate

Superscript

$(\sim)$	dimensionless

Subscripts

e	early regime state
s	late regime state

The constructal tree-networks are deterministic because every single feature of the network is determined, neither guessed nor assumed [12]. They start from a given small finite scale, and they are assembled as confluences of channels to other longer and larger channels paths that in turn merge to even longer and larger channels until the system volume is filled. The resulting relationships of length ratio, width or diameter ratio, number of lower level constructs are determined for every given condition.

The smallest scale is the one in which the first channel occurs. This is the elemental system, where the flow turns from slow and volumetrically to fast and aligned thus making the connection of an infinity number of points to a finite entity (e.g., [7,13]).

In all constructal publications so far the elemental system has been defined as a finite size rectangle [1] or circular sector [8] with insulated or impermeable boundaries except of a single outlet. The level-0 channel has been placed in the symmetric position. As hypothesis, that works because it can be shown that the position that would facilitate the flow the most would be centered. On the other hand, can the elemental channel, hence the constructal element, evolve from an even simpler structure?

In this paper we draw attention to the formation of the elemental system. We start with an even simpler configuration than the literature does (Fig. 1). We then let it evolve by itself under the Constructal Law: given freedom to morph, the design of the system will evolve to facilitate the flow of the currents that pass through it. The constructal element will progressively have its thermal resistance reduced. Therefore a history of the evolution of the design will be determined. It starts with the simplest configuration (Fig. 2): a rectangular elemental system with fixed area (A), fixed shape (H_0/L_0), as well as a fixed width of the outlet port (D) through which heat is conveyed to the surroundings. Inside the system there is a relative high-conductivity squared insert that is free to evolve its shape (h_x, h_y), its position (x_c, y_c) and area (A_p).

The evolution of the internal configuration or the design of the system takes place by a Darwinian-like process, a version of today's method called evolutionary computing (e.g., [21]). A finite size progeny is randomly generated based on the traits of one parent, and then a *fitness* condition is applied to all the population. One individual or a group is selected to pass the traits to the next generation. In our paper the traits are determined by four parameters of the high conductivity insert and only one configuration (individual) survives. The evolutionary algorithm proceeds until an established limit.

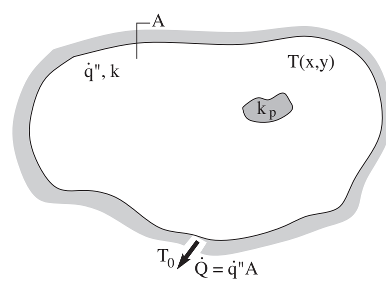

Fig. 1. Conceptual organism of known volume $A \times W$ ($W \ll A^{1/2}$), with a gray patch that does not perform its main function $\dot{q}'''$ but conducts heat more easily.

This paper considers the very first CT application [12], which was on conducting paths for cooling a heat generating volume. The fitness condition is the overall thermal resistance across the system. The condition was computed by the Finite Element Method (FEM).

Results showed that (a) the elemental system evolves to configuration similar to ones customarily assumed in CT, but possibly with shorter length leaving a gap at the far end; (b) the resulting configurations are almost path independent and; (c) the gain in performance resembles an S curve, which is explained theoretically.

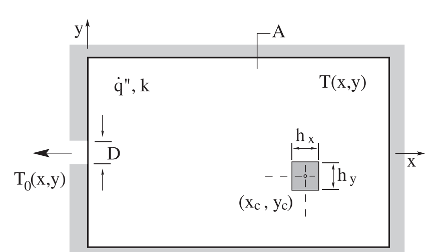

Fig. 2. A special case of the system of Fig. 1 with fixed rectangular shape H_0/L_0 that undergoes evolution of its internal configuration expressed by the set $\{\tilde{x}_c, \tilde{y}_c, \tilde{h}_x, \tilde{h}_y\}$.